Model-based Probabilistic Planning

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Goal

a brief introduction to various goal-oriented MDPs

an extensive discussion of heuristic search

a discussion on connections with classical planning

Plan

- Lecture 10
  - Definition of Stochastic Shortest Path
  - Various Heuristic Search Algorithms for SSPs
- Lecture 11
  - Extensions of SSPs for MDPs with Dead Ends
  - Determinization-based Approximation Algorithms

Our Setting

- vs. RL (Zico): model of the world is known
- vs. flat: model of the world in a declarative representation
  - symbolic
  - large problems
- vs. reward (Scott): goal directed
  - PPDDL vs RDDL
- vs. finite-horizon MDPs (Thomas): indefinite horizon
- vs. classical planning (Malte/Gabi): probabilities
- vs. complete state space: knowledge of the start state
- domain independent: no additional human input

Heuristic Search for SSPs

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Infinite Horizon Discounted Reward MDP

- S: A set of states
- A: A set of actions
- T(s,a,s'): transition model
- R(s,a,s'): reward
- γ: discount factor

Where Does γ Come From?

- **γ can affect optimal policy significantly**
  - $\gamma = 0 + \varepsilon$: yields myopic policies for "impatient" agents
  - $\gamma = 1 - \varepsilon$: yields far-sighted policies, inefficient to compute

- How to set it?
  - Sometimes suggested by data
    - (e.g., inflation or interest rate)
  - Often set to whatever gives a reasonable policy

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Stochastic Shortest Path MDP

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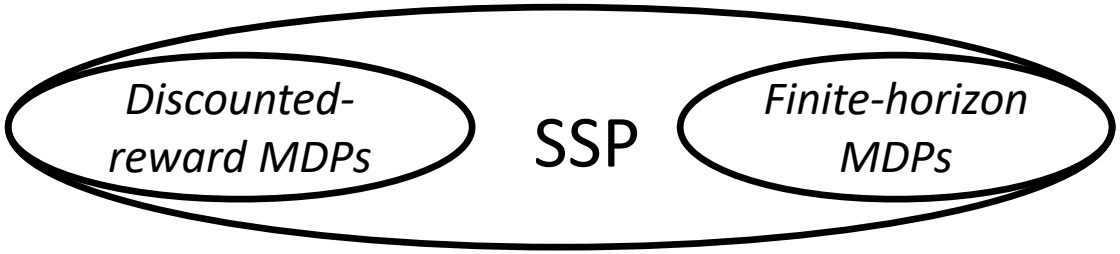
Minimize
- expected cost to reach a goal
- under full observability
- indefinite horizon

Bellman Equations for SSP

$$
\begin{aligned}
V^*(s) &= 0 \qquad \text{if } s \in \mathcal{G} \\
&= \min_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{T}(s, a, s') \left[\mathcal{C}(s, a, s') + V^*(s') \right]
\end{aligned}
$$

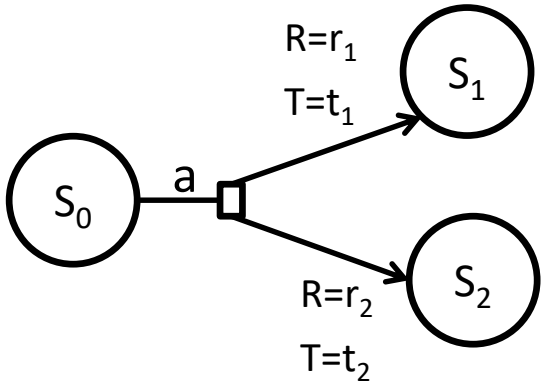
add base case; no discount factor

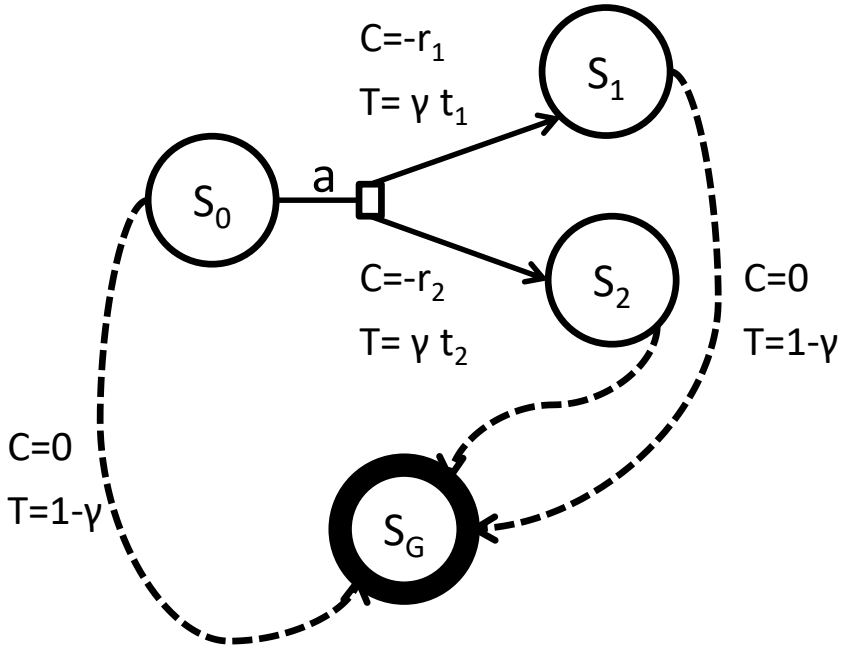
SSP vs. IHDR?



Discounted Reward MDP → SSP

[Bertsekas&Tsitsiklis 95]





When is SSP well formed/defined

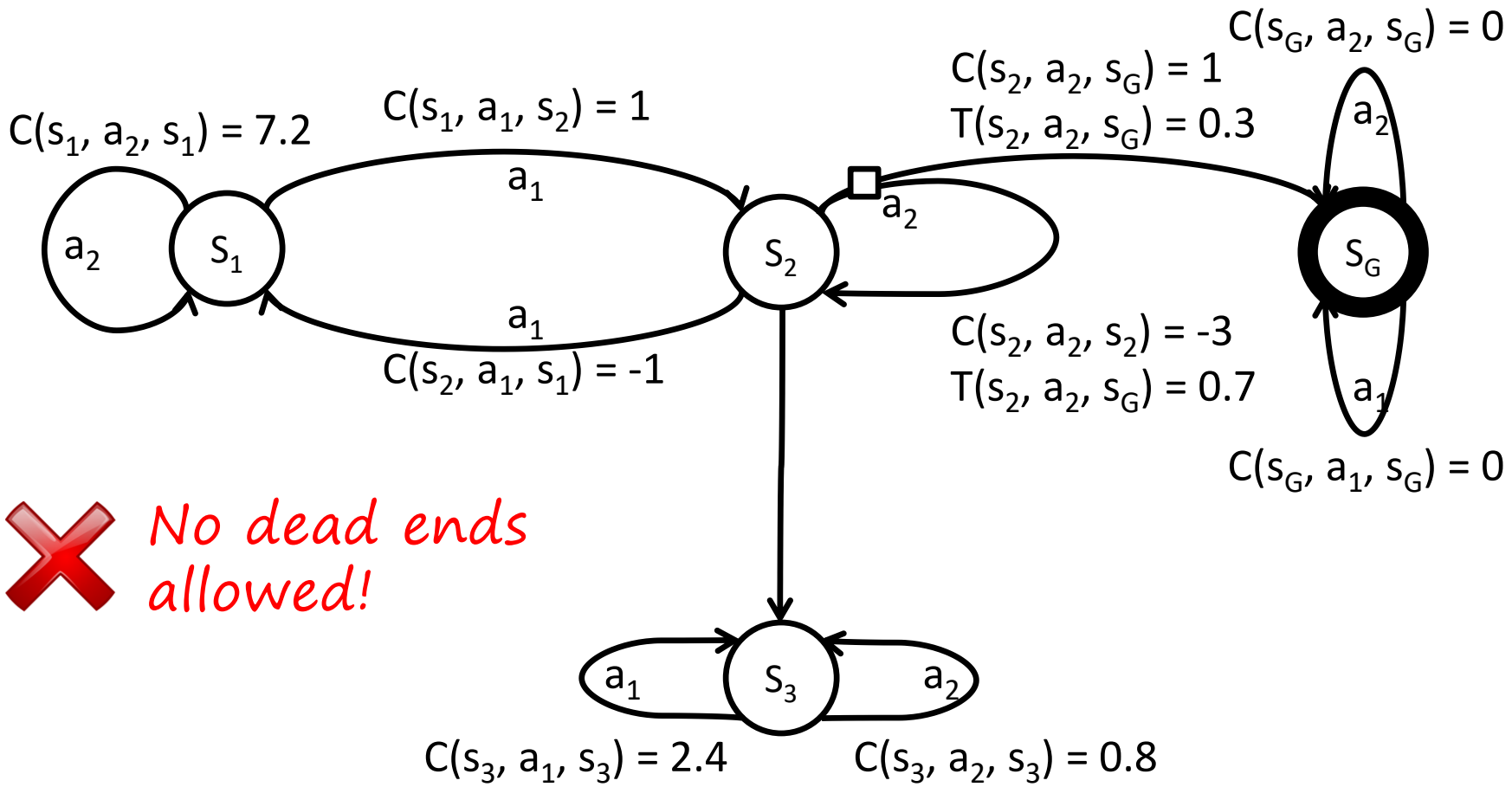
[Bertsekas, 1995]

- S: A set of states
- A: A set of actions
- T(s,a,s'): transition model
- C(s,a,s'): cost
- G: set of goals

Under two conditions:

- There is a *proper policy* (reaches a goal with P=1 from all states)
- Every *improper policy* incurs a cost of ∞ from every state from which it does not reach the goal with P=1

Not an SSP MDP Example



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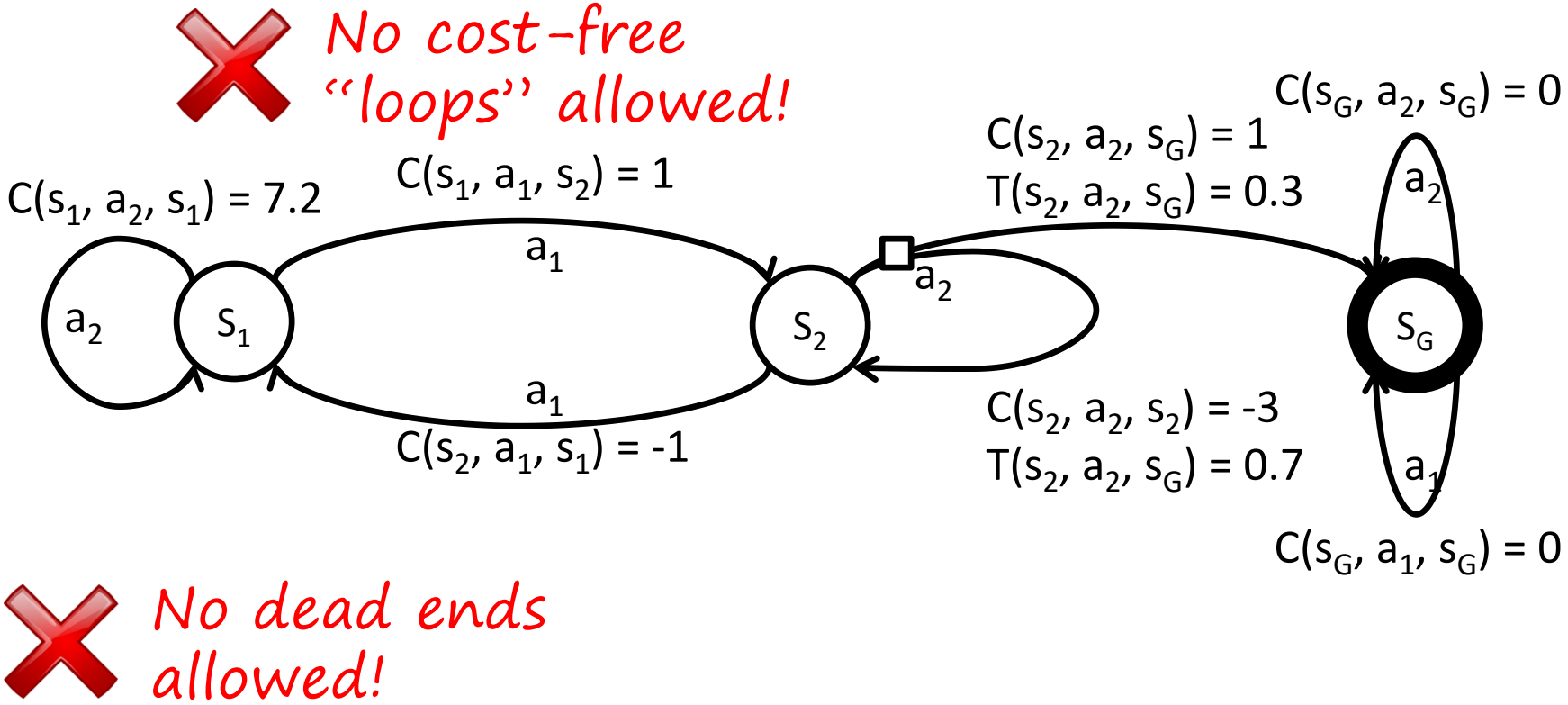


Value Iteration [Bellman 57]

No restriction on initial value function

iteration n

1 initialize V_0 arbitrarily for each state
2 $n \leftarrow 0$
3 **repeat**
4 $\quad n \leftarrow n+1$
5 $\quad$ **foreach** $s \in \mathcal{S}$ **do**
6 $\quad\quad$ compute $V_n(s)$ using Bellman backup at s
7 $\quad\quad$ compute $\text{residual}_n(s) = |V_n(s) - V_{n-1}(s)|$
8 $\quad$ **end**
9 **until** $\max_{s \in \mathcal{S}} \text{residual}_n(s) < \epsilon$;
10 return greedy policy: $\pi^{V_n}(s) = \text{argmin}_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{T}(s,a,s') \left[\mathcal{C}(s,a,s') + V_n(s')\right]$

ϵ-consistency

termination condition

VI → Asynchronous VI

- Is backing up ***all*** states in an iteration essential?
  - No!
- States may be backed up
  - as many times
  - in any order
- If no state gets starved
  - convergence properties still hold!!

Residual wrt Value Function V (Res^V)

- Residual at *s* with respect to *V*
  - magnitude($\Delta V(s)$) after one Bellman backup at s

$$Res^V(s) = \left| V(s) - \min_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{T}(s,a,s')[\mathcal{C}(s,a,s') + V(s')] \right|$$

- Residual wrt respect to *V*
  - max residual
  - $Res^V = \max_s (Res^V(s))$

$Res^V < \epsilon$ (ϵ-consistency)

(General) Asynchronous VI

1 initialize V arbitrarily for each state
2 **while** $Res^V > \epsilon$ **do**
3 | find a state s
4 | revise $V(s)$ using a Bellman backup at s
5 | update $Res^V(s)$
6 **end**
7 return greedy policy π^V

Asynch VI: Lots of Extensions to VI

- Prioritized Sweeping
  - select s that is likely to have the most change in V
- Backward VI
  - backup states in reverse order starting from goal
- Partitioned VI
  - divide states into partitions
  - backup partitions in reverse order from goal partition

Heuristic Search Algorithms

- Definitions
- Find & Revise Scheme.
- LAO* and Extensions
- RTDP and Extensions
- Other uses of Heuristics/Bounds
- Heuristic Design

Limitations of VI/Extensions

- Scalability
  - Memory linear in size of state space
  - Time at least polynomial or more
- Polynomial is good, no?
  - state spaces are usually huge.
  - if *n* state vars then 2^n states!
- Curse of Dimensionality!

Heuristic Search

- Insight 1
  - knowledge of a start state to save on computation
    ~ (all sources shortest path → single source shortest path)

- Insight 2
  - additional knowledge in the form of heuristic function
    ~ (dfs/bfs → A*)

Model: SSP_{s0}

- S: A set of states
- A: A set of actions
- T(s,a,s'): transition model
- C(s,a,s'): cost
- G: set of goals
- s_0: start state

Under two conditions:

- There is a *proper policy* (reaches a goal with P= 1 from all states)
- Every *improper policy* incurs a cost of ∞ from every state from which it does not reach the goal with P=1

Model

- SSP (as before) with an additional start state s_0
  - denoted by SSP_{s0}

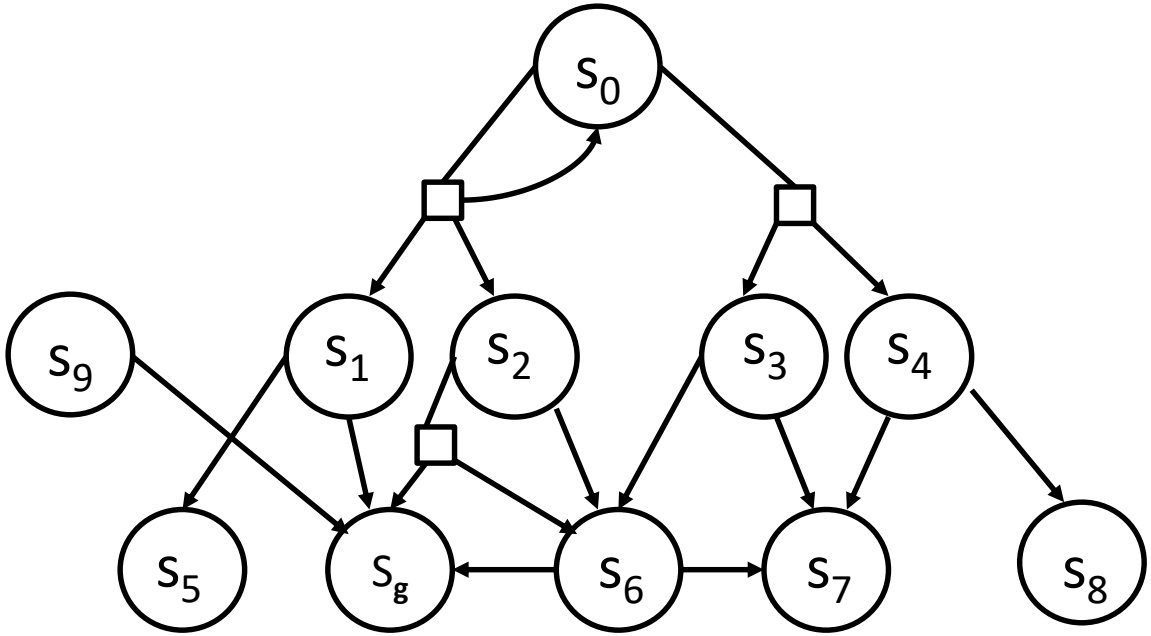
- What is the solution to an SSP_{s0}
- Policy ($S \rightarrow A$)?
  - are states that are not reachable from s_0 relevant?
  - states that are never visited (even though reachable)?

Partial Policy

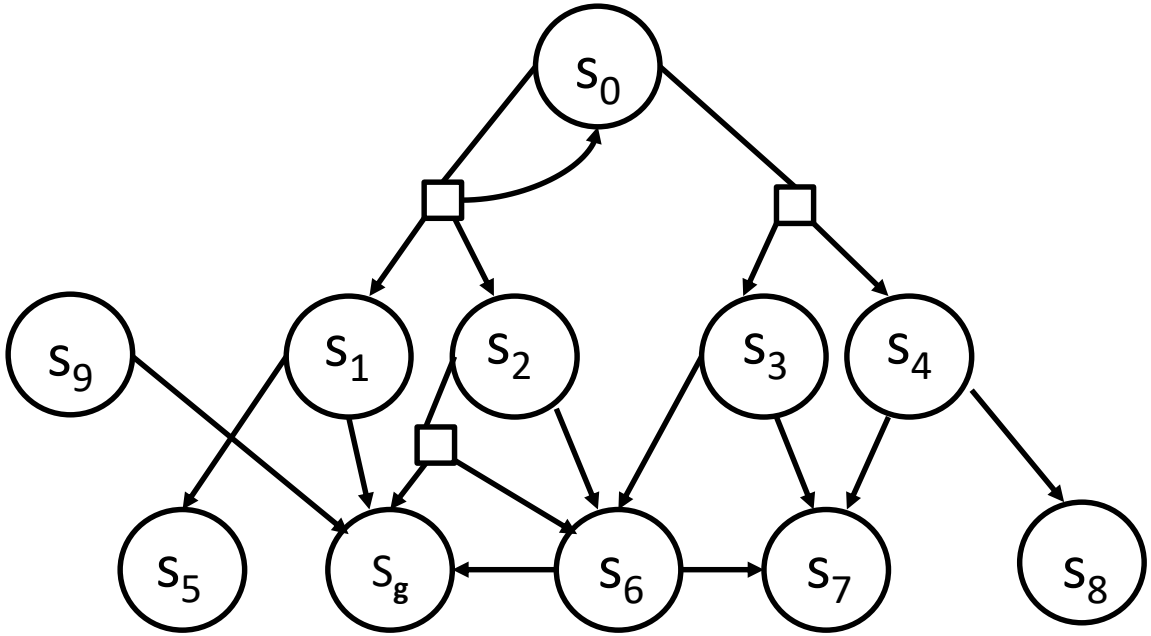
- Define *Partial policy*
  - π: S' $\rightarrow$ *A*, where S'$\subseteq$ *S*

- Define *Partial policy closed w.r.t. a state s.*
  - is a partial policy π_s
  - defined for all states *s'* reachable by π_s starting from *s*

Partial policy closed wrt s_0



Partial policy closed wrt s_0



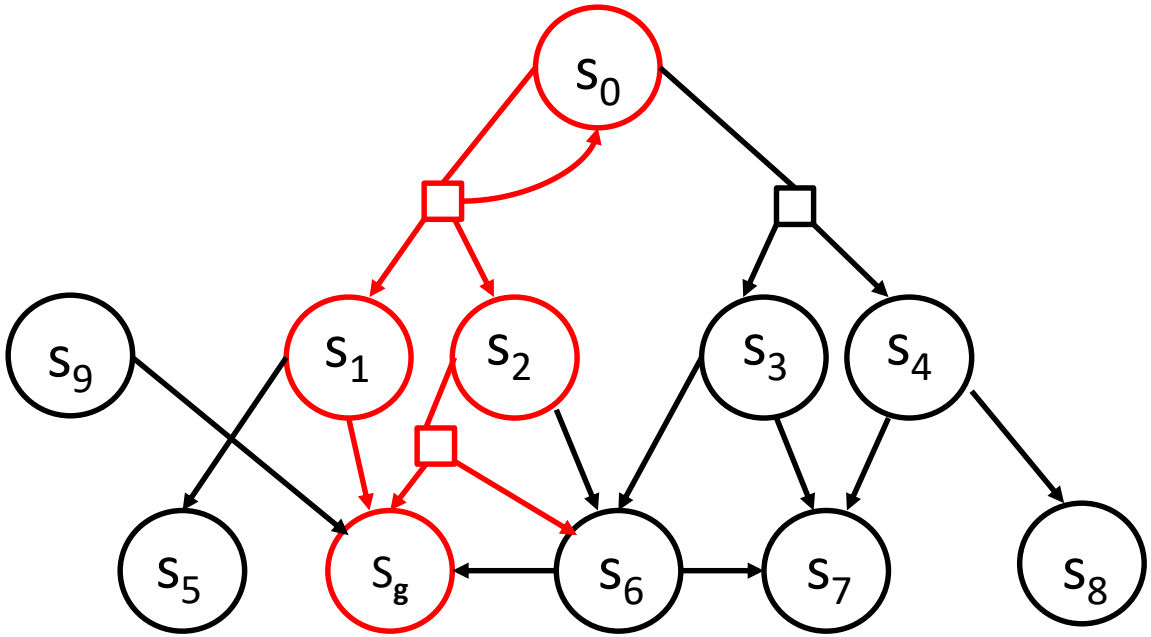
Is this policy closed wrt s_0?

$\pi_{s0}(s_0) = a_1$

$\pi_{s0}(s_1) = a_2$

$\pi_{s0}(s_2) = a_1$

Partial policy closed wrt s_0



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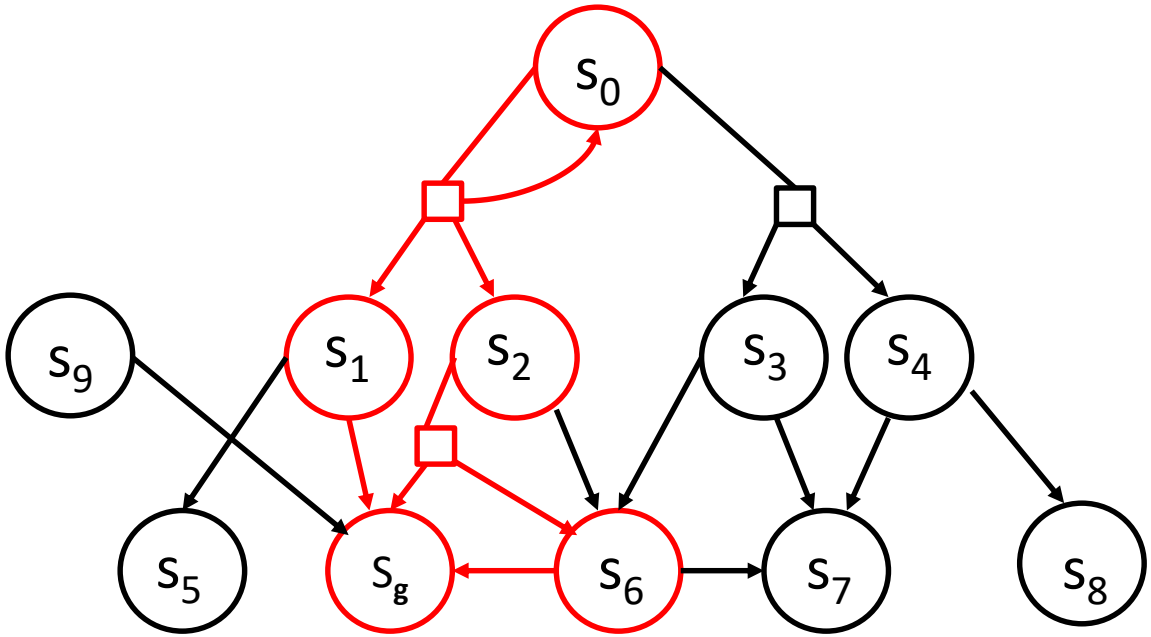
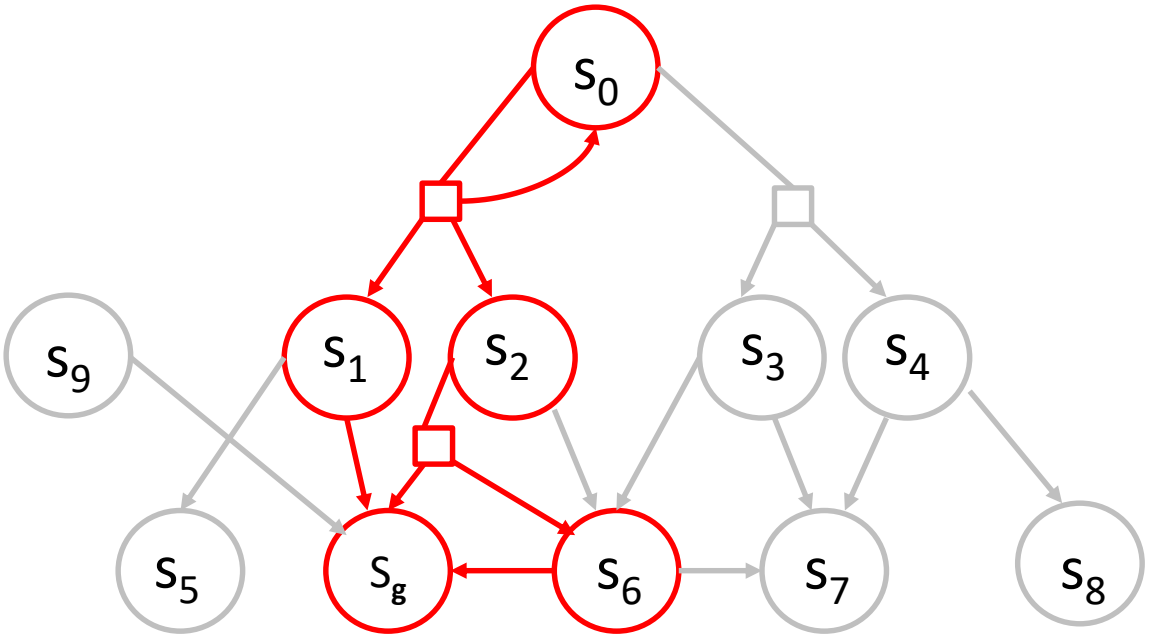


Is this policy closed wrt s_0?

$\pi_{s0}(s_0) = a_1$
$\pi_{s0}(s_1) = a_2$
$\pi_{s0}(s_2) = a_1$
$\pi_{s0}(s_6) = a_1$

Policy Graph of π_{s0}



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Greedy Policy Graph

- Define *greedy policy*: π^V = argmin$_a$ Q^V(s,a)
- Define *greedy partial policy rooted at s_0*
  - Partial policy rooted at s_0
  - Greedy policy
  - denoted by π^V_{s0}
- Define *greedy policy graph*
  - Policy graph of π^V_{s0} : denoted by G^V_{s0}

Heuristic Function

- h(s): $S \rightarrow \mathbb{R}$
  - estimates V*(s)
  - gives an indication about "goodness" of a state
  - usually used in initialization $V_0(s) = h(s)$
  - helps us avoid seemingly bad states

- Define *admissible* heuristic
  - optimistic
  - $h(s) \leq V^*(s)$

Heuristic Search Algorithms

- Definitions
- Find & Revise Scheme.
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- RTDP and Extensions
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A General Scheme for Heuristic Search in MDPs

- Two (over)simplified intuitions
  - Focus on states in greedy policy wrt V rooted at s_0
  - Focus on states with residual > ϵ
- Find & Revise:
  - repeat
    - find a state that satisfies the two properties above
    - perform a Bellman backup
  - until no such state remains

FIND & REVISE [Bonet&Geffner 03a]

1 Start with a heuristic value function $V \leftarrow h$
2 **while** *V's greedy graph $G^V_{s_0}$ contains a state s with $Res^V(s) > \epsilon$* **do**
3 | FIND a state s in $G^V_{s_0}$ with $Res^V(s) > \epsilon$
4 | REVISE *V(s)* using a Bellman backup at s
5 **end**
6 return a π^V

- Convergence to V* is guaranteed
  - if heuristic function is admissible
  - ~no state gets starved in ∞ FIND steps

Heuristic Search Algorithms

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LAO* family

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand some states on the fringe (in greedy graph)
- initialize all new states by their heuristic value
- choose a subset of affected states
- perform some REVISE computations on this subset
- recompute the greedy graph

until greedy graph has no fringe & residuals in greedy graph small

output the greedy graph as the final policy

LAO* [Hansen&Zilberstein 98]

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand best state *s* on the fringe (in greedy graph)
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- perform VI on this subset
- recompute the greedy graph

until greedy graph has no fringe ~~& residuals in greedy graph small~~

output the greedy graph as the final policy

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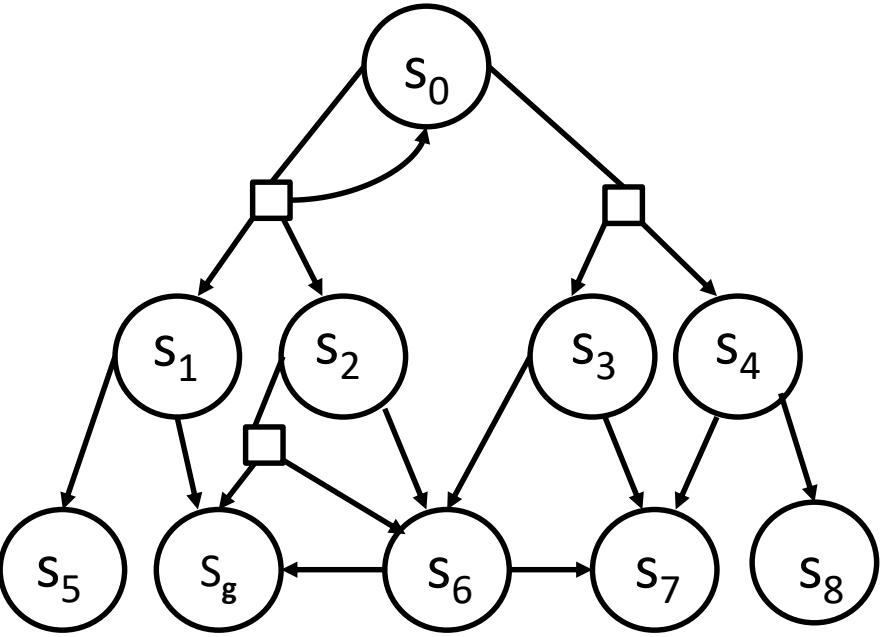
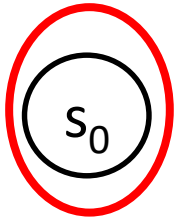
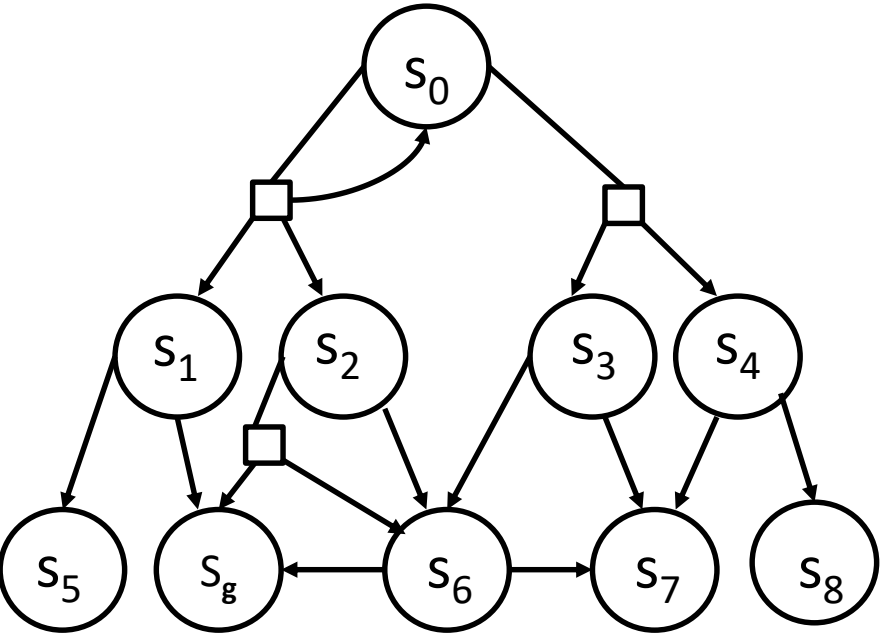


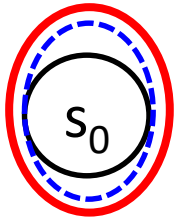


$V(s_0) = h(s_0)$

add s_0 in the fringe and in greedy graph

LAO*





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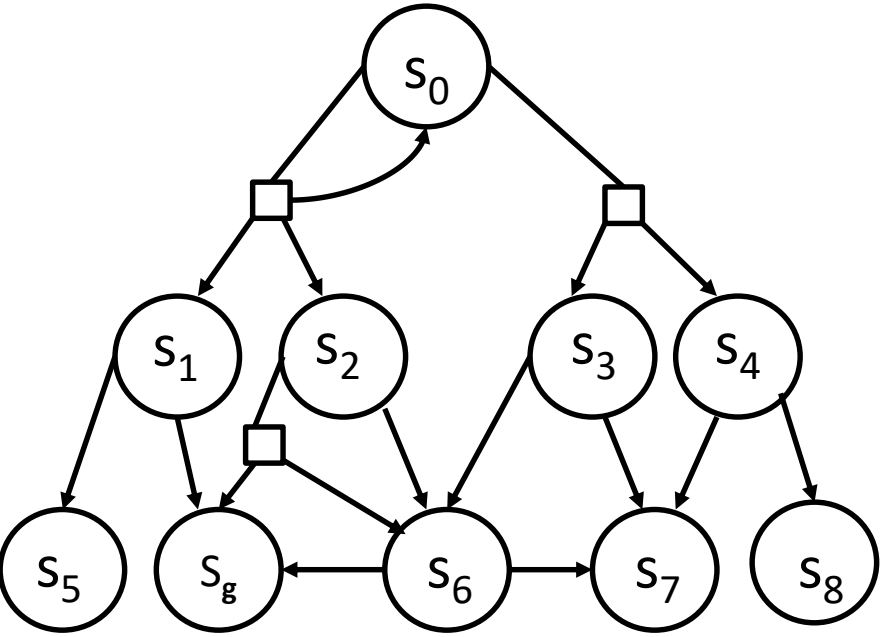
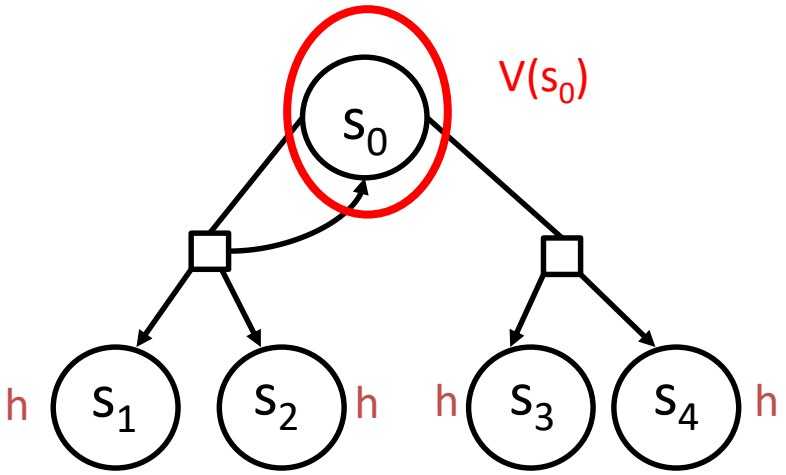
FIND: expand some states on the fringe (in greedy graph)

LAO*





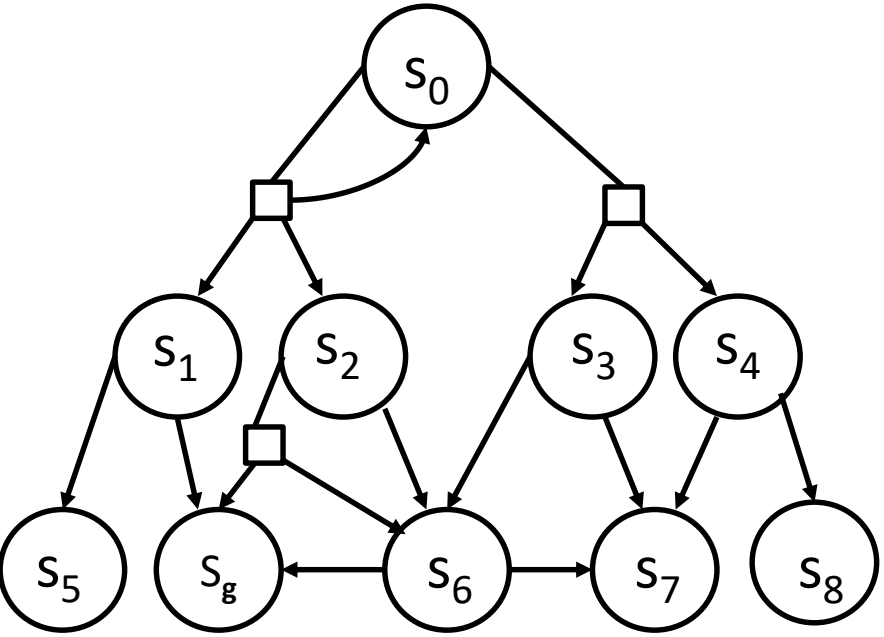
FIND: expand some states on the fringe (in greedy graph)

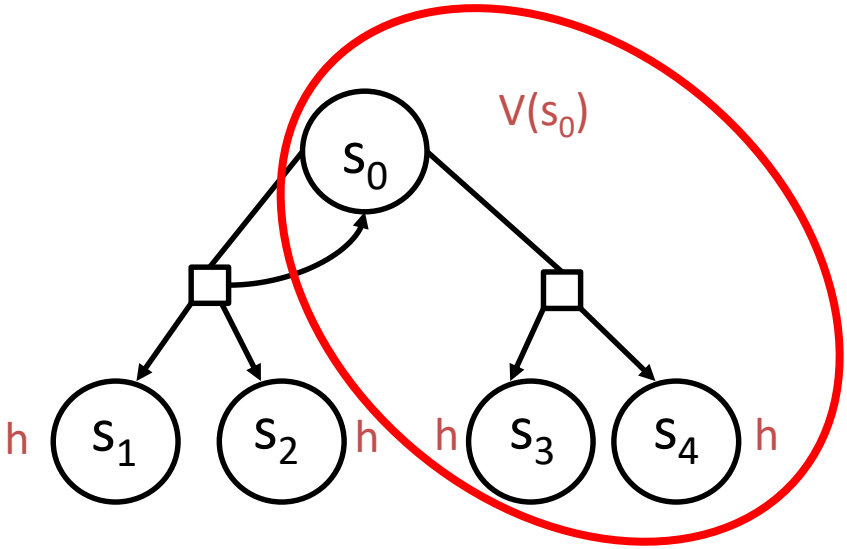
initialize all new states by their heuristic value

subset = all states in expanded graph that can reach s

perform VI on this subset

LAO*





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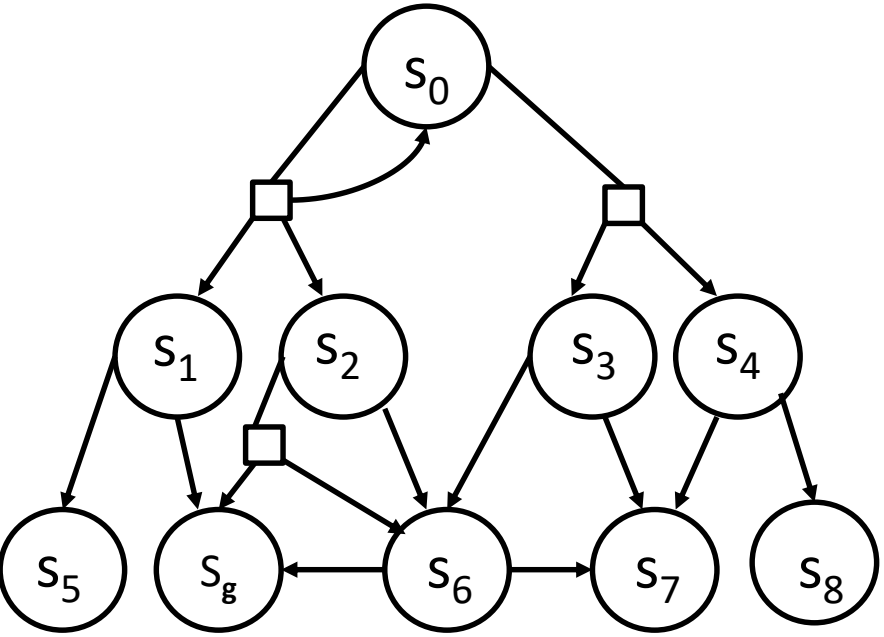

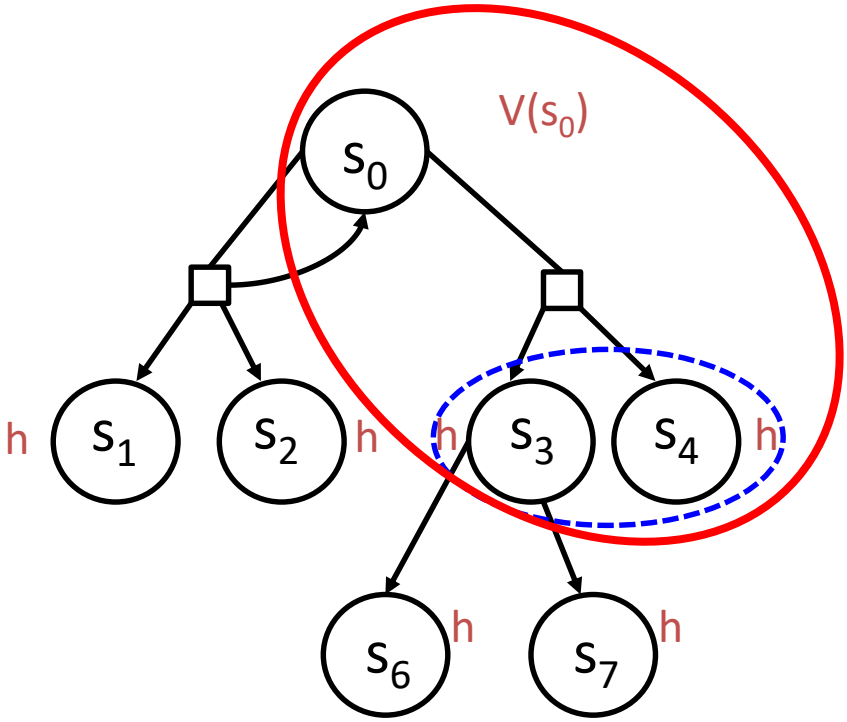

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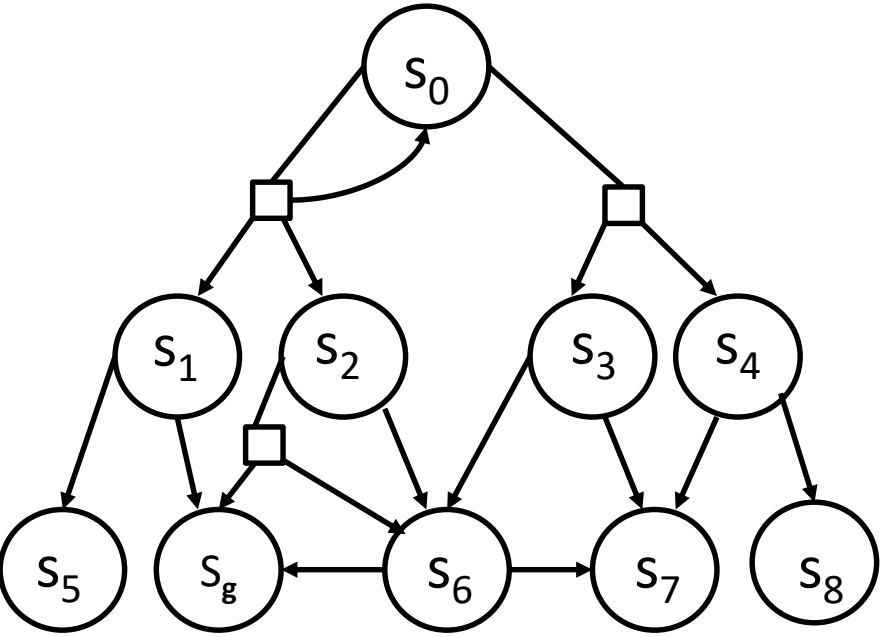

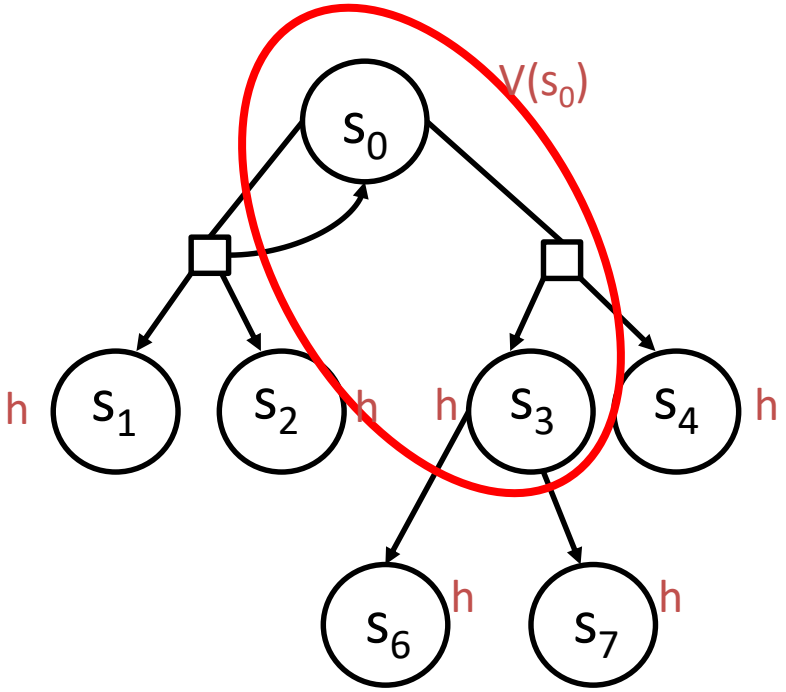

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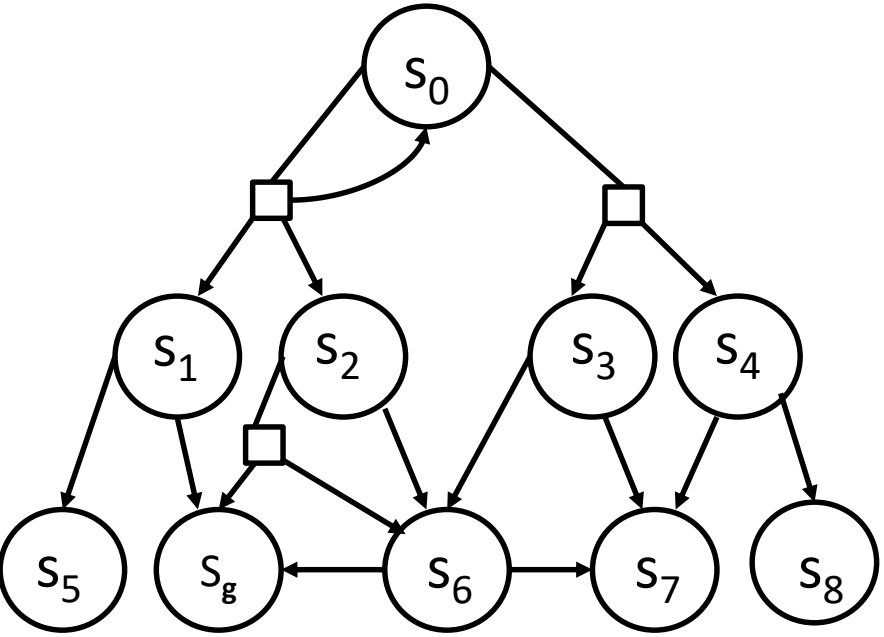

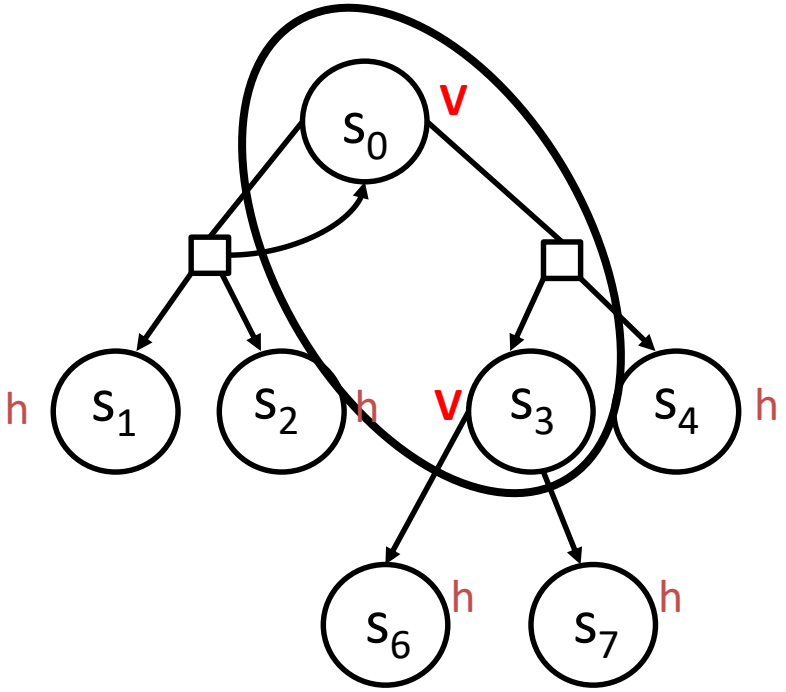

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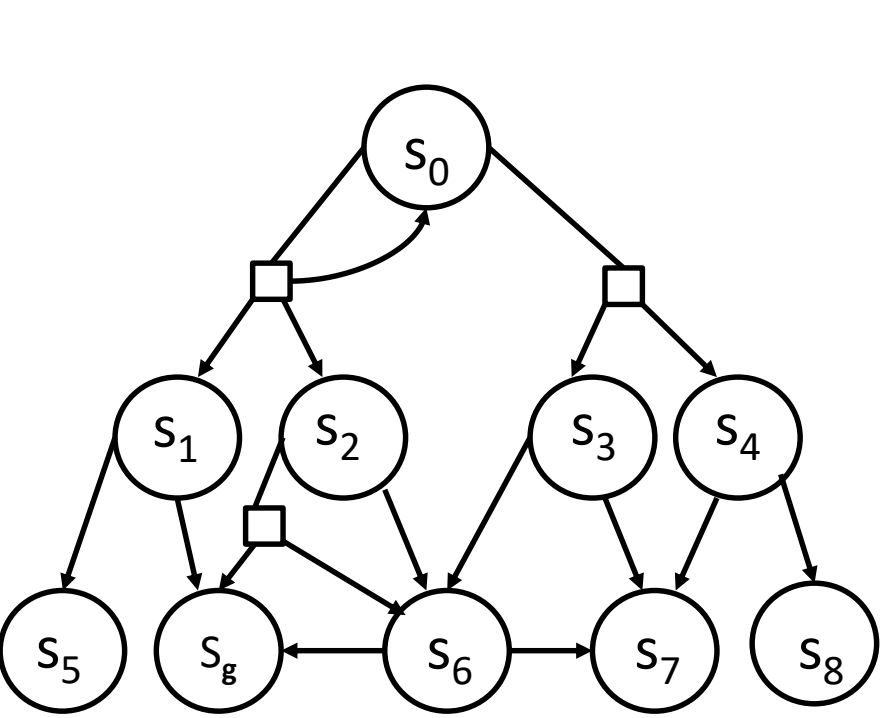

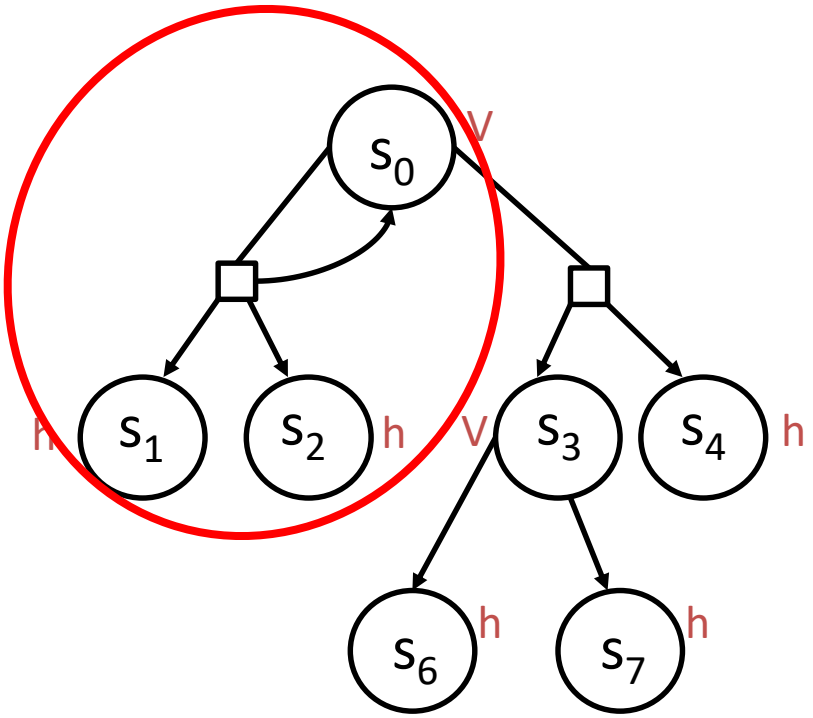

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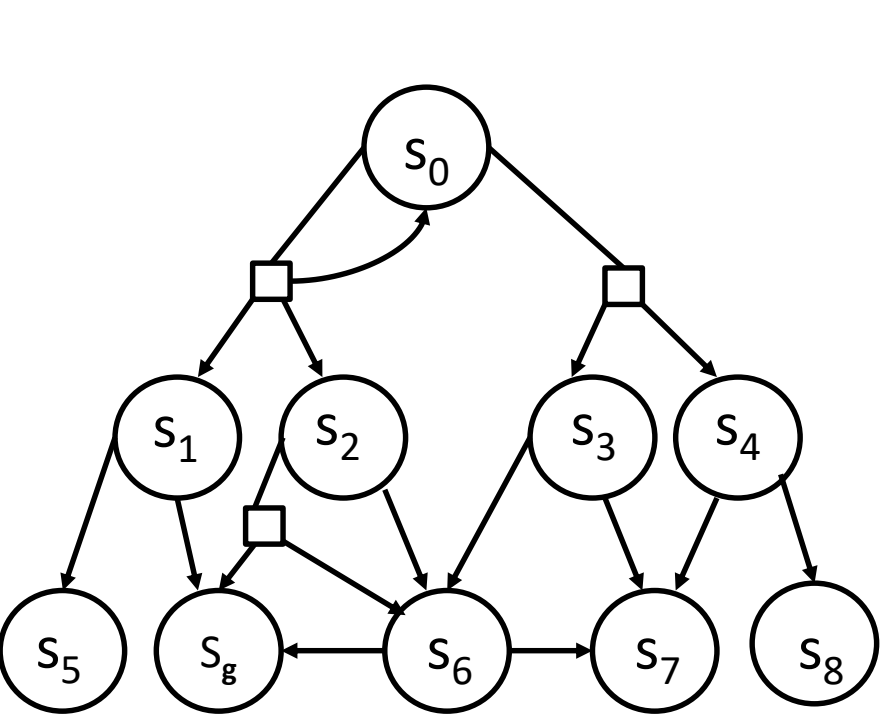

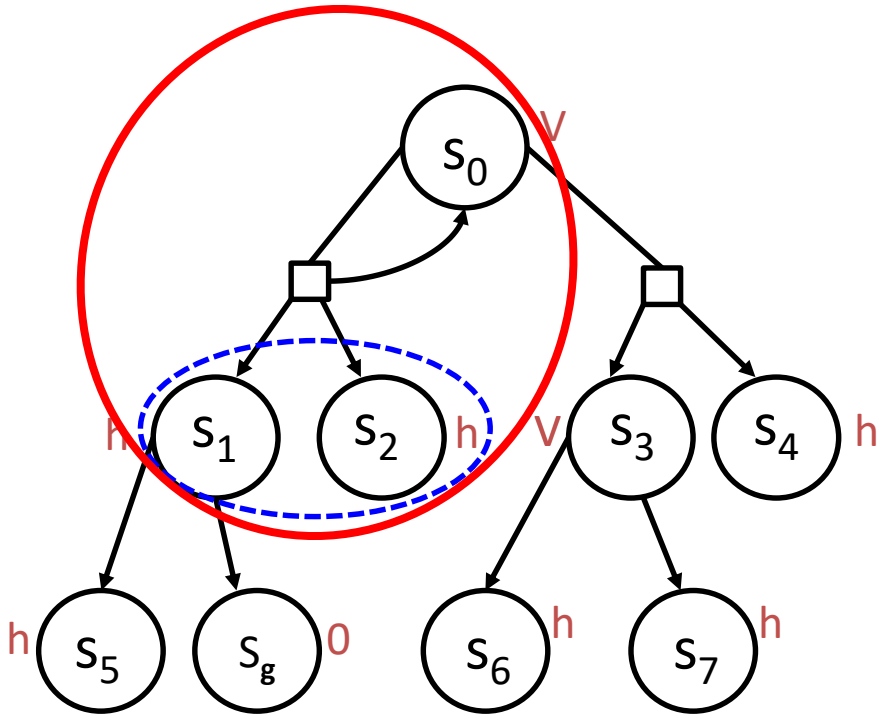

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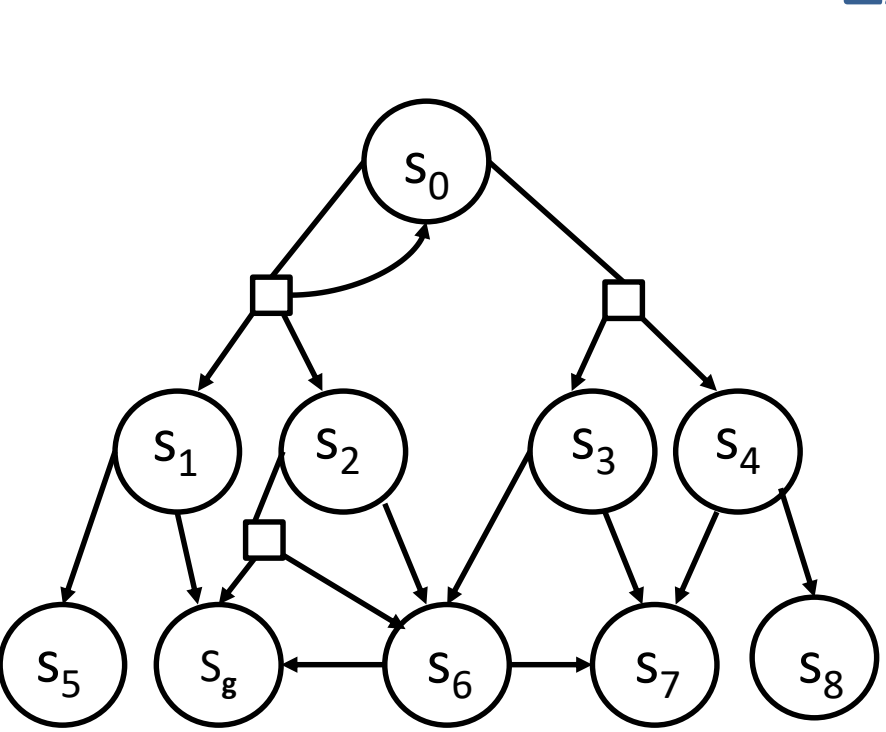

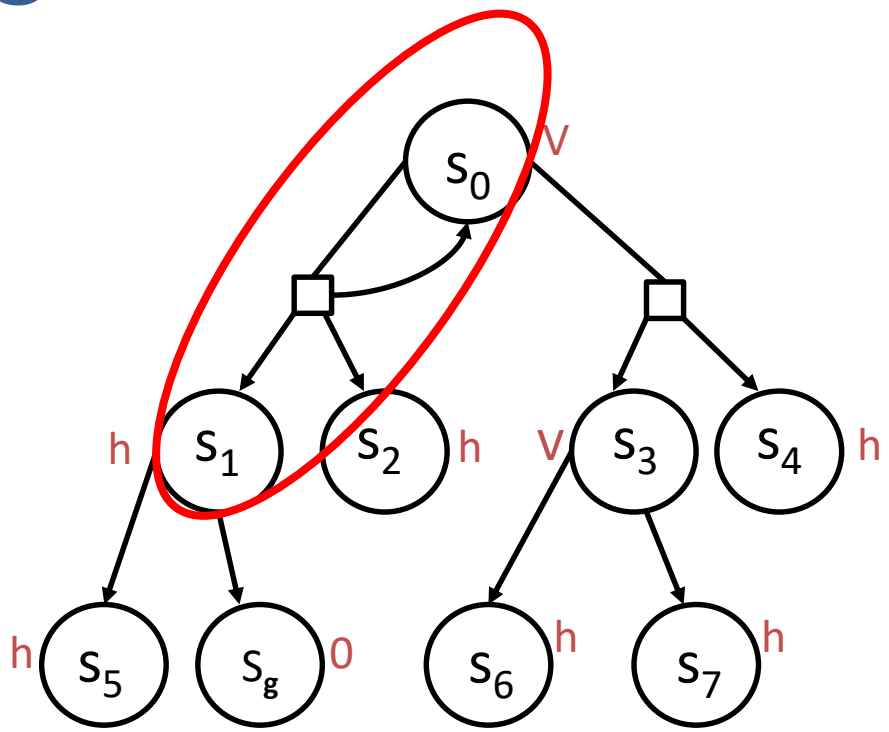

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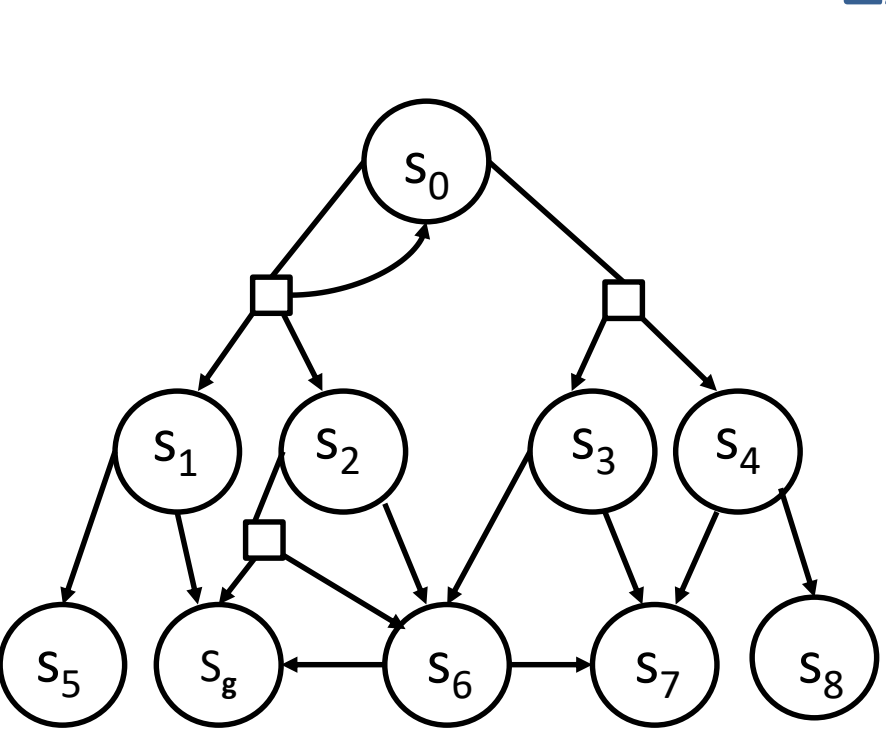

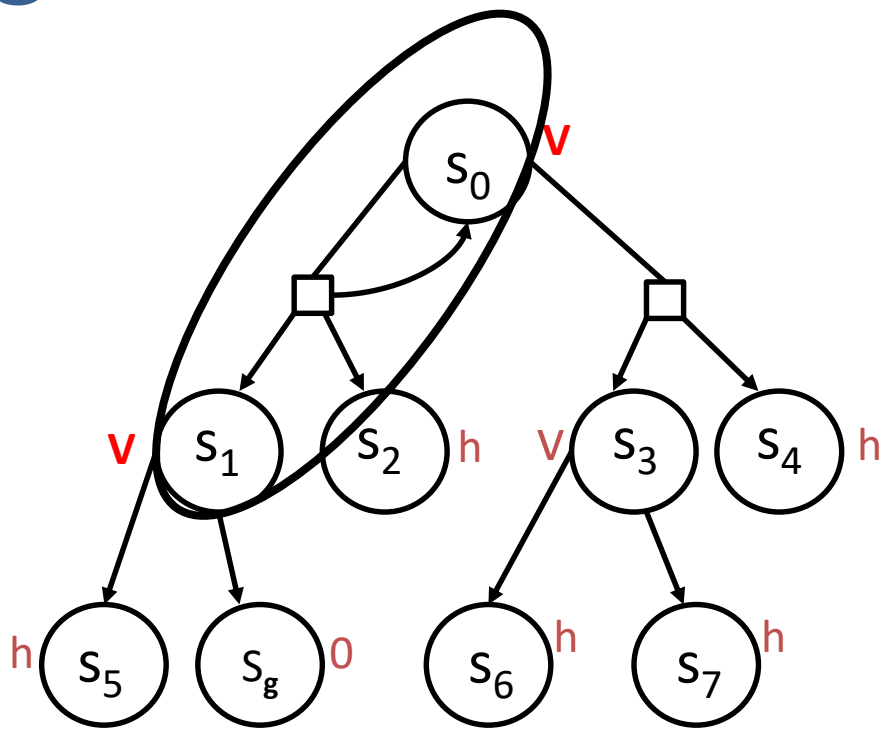

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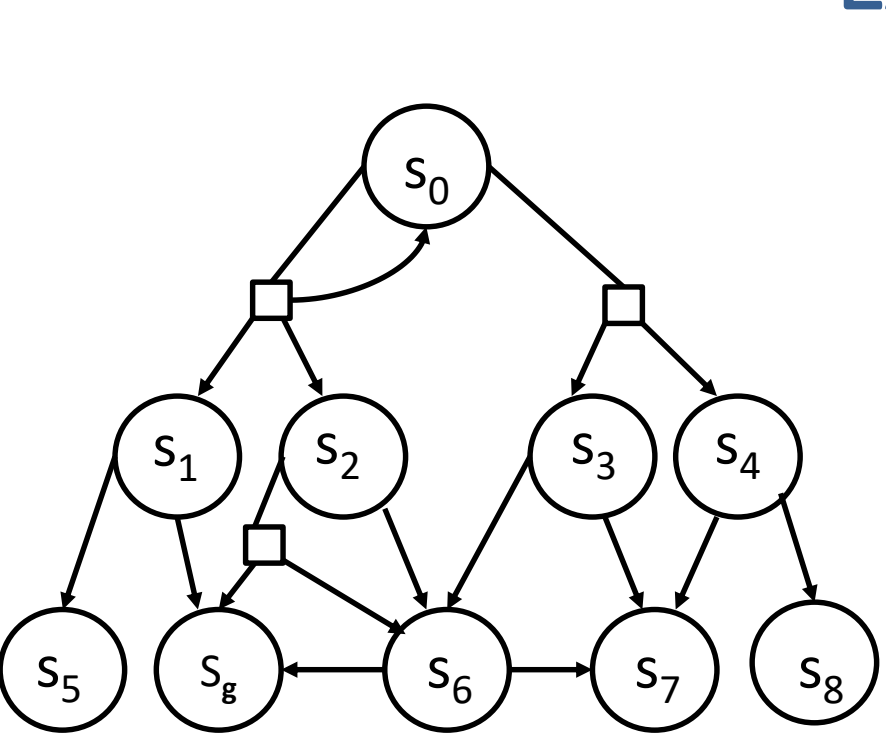

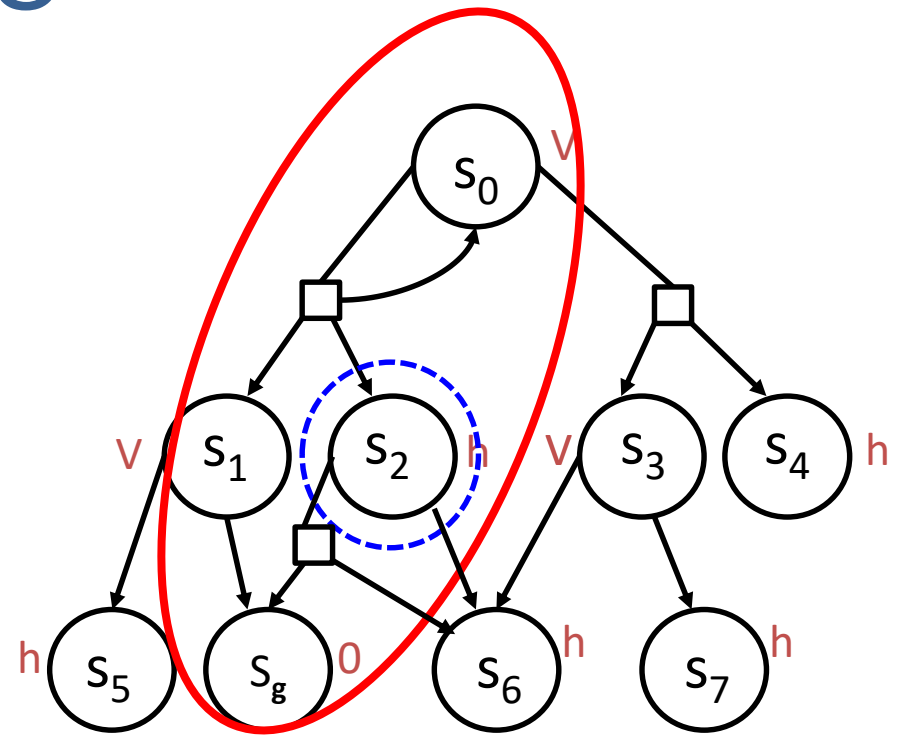

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initialize all new states by their heuristic value

subset = all states in expanded graph that can reach s

perform VI on this subset

recompute the greedy graph

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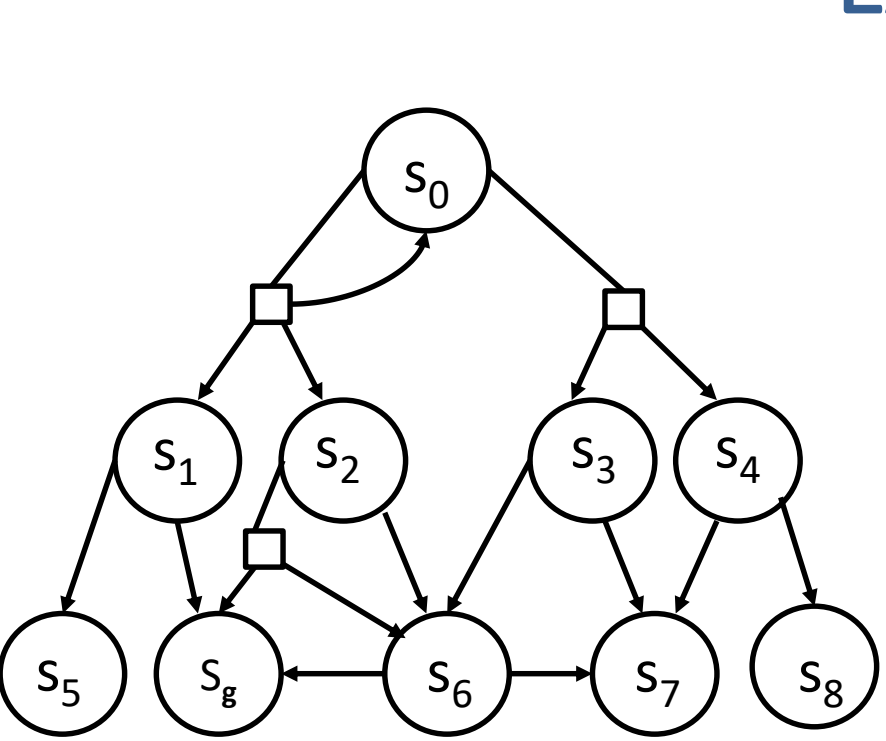

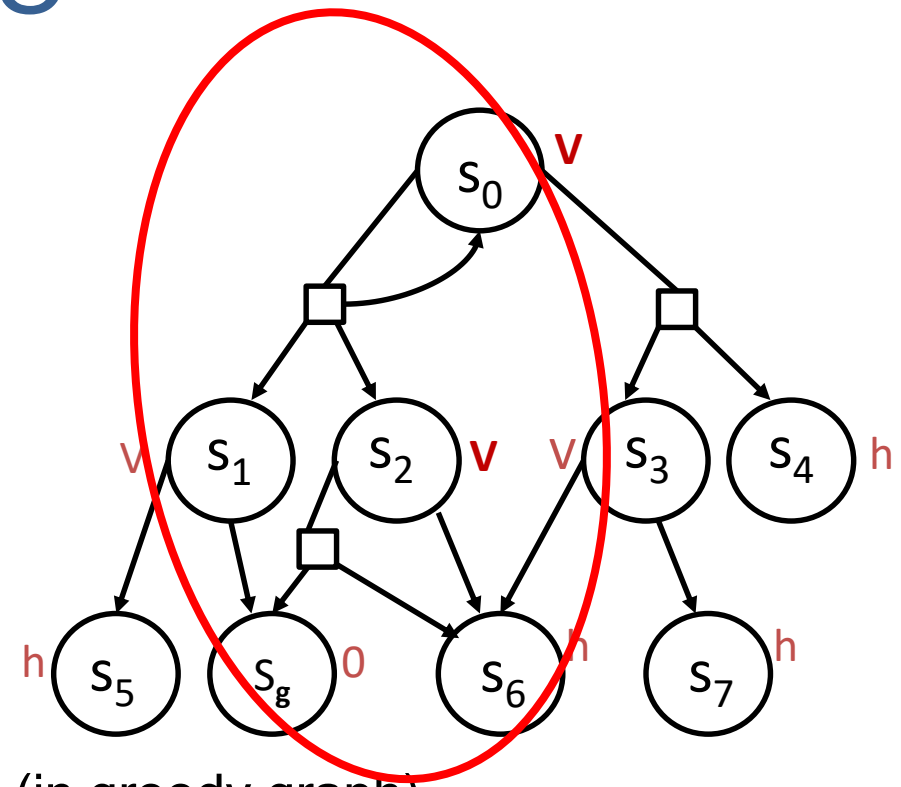

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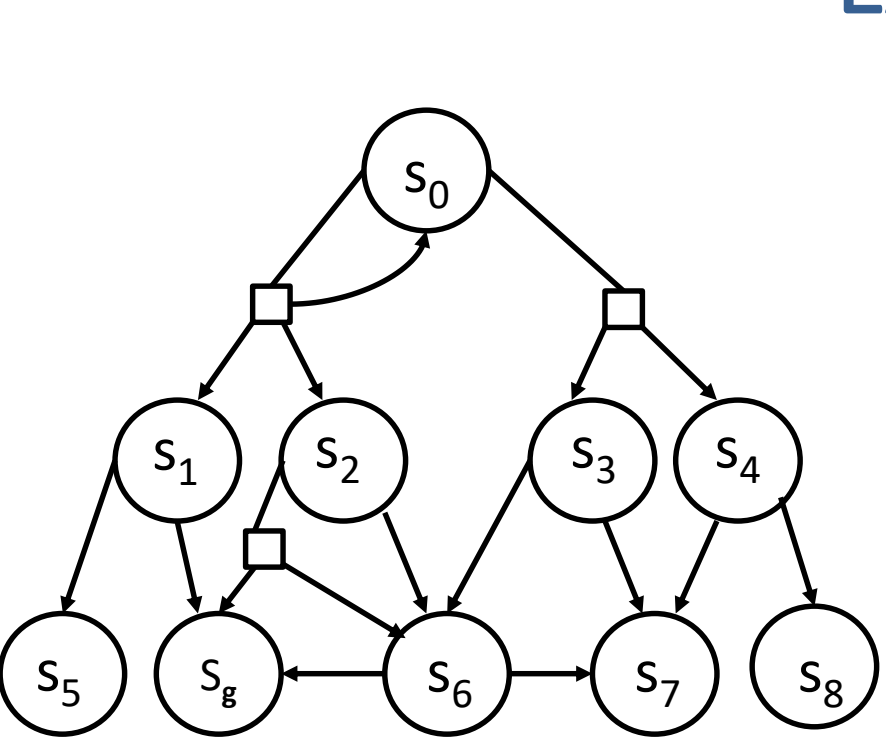

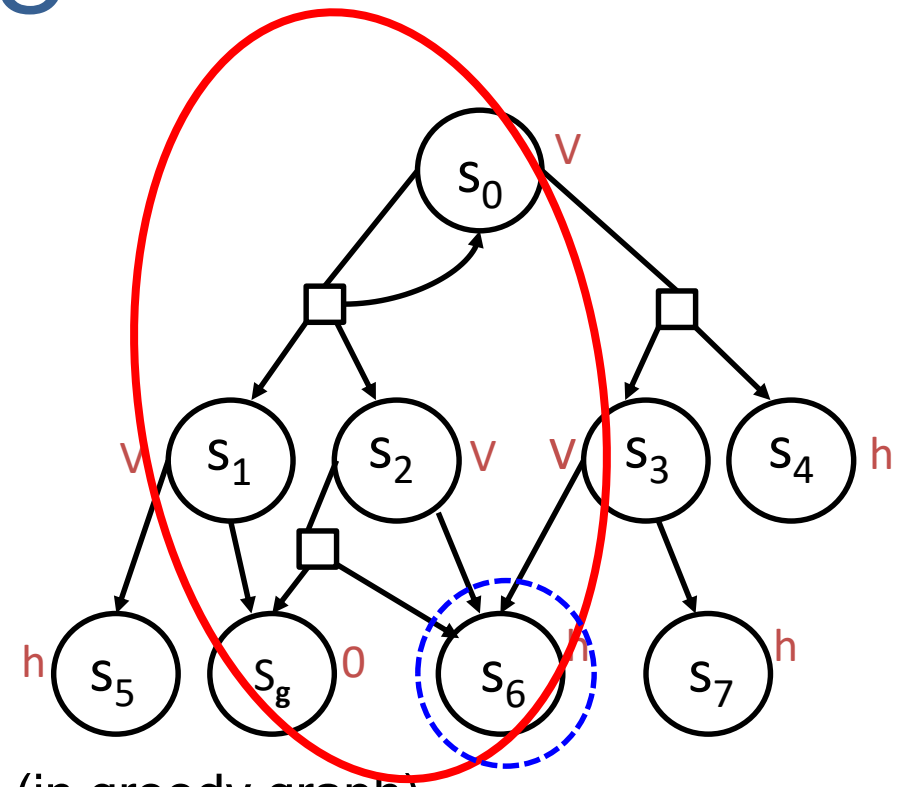

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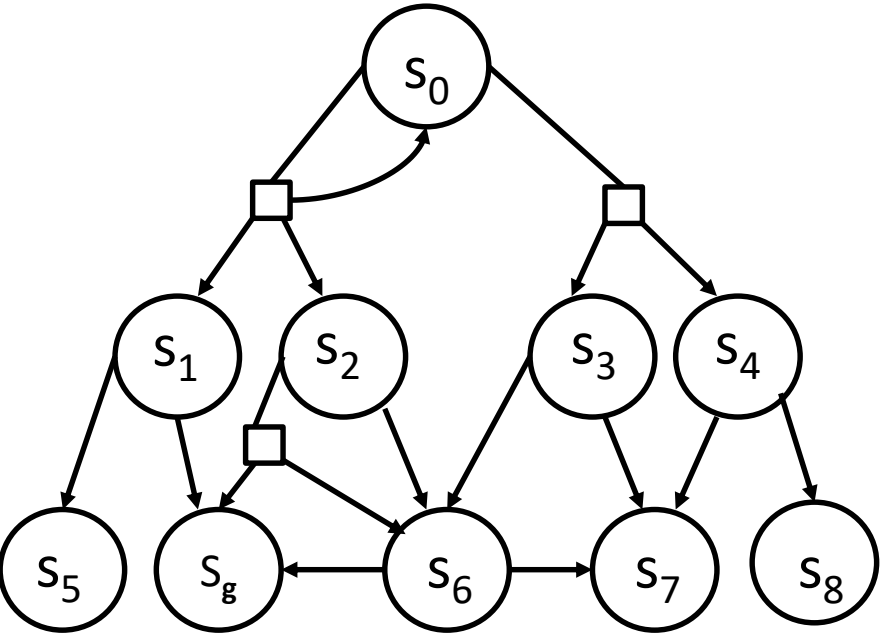
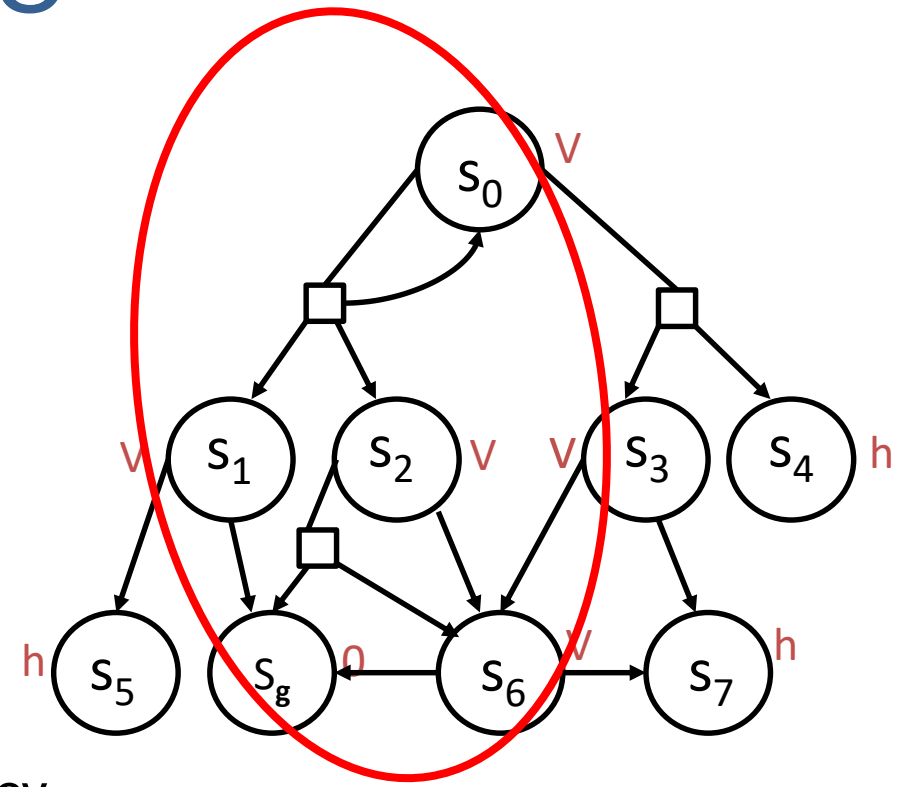
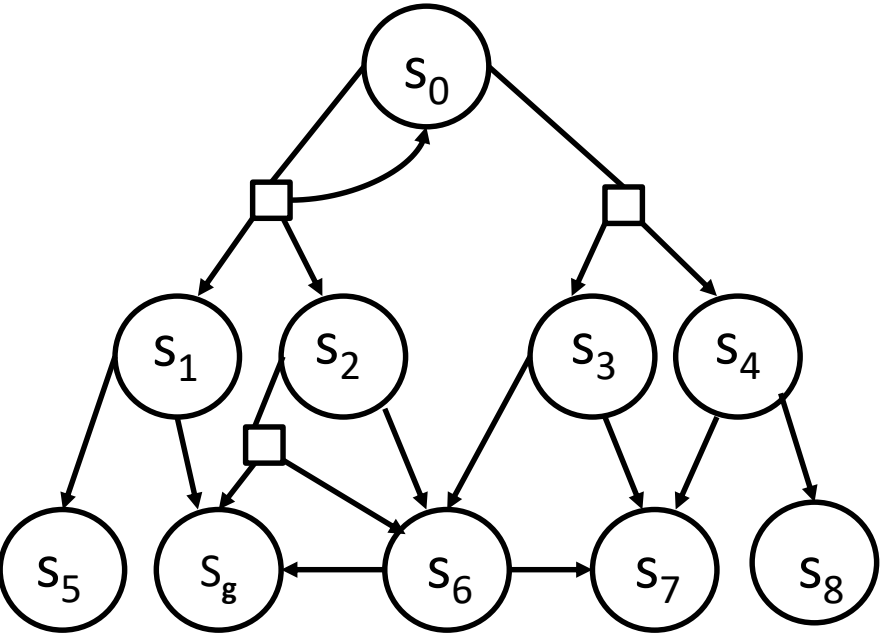
LAO*

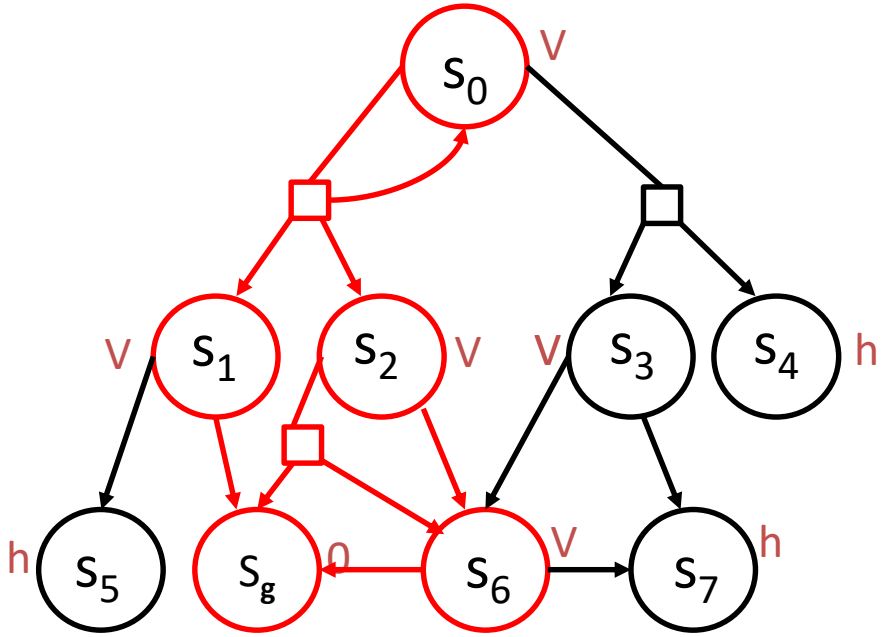




output the greedy graph as the final policy

LAO*





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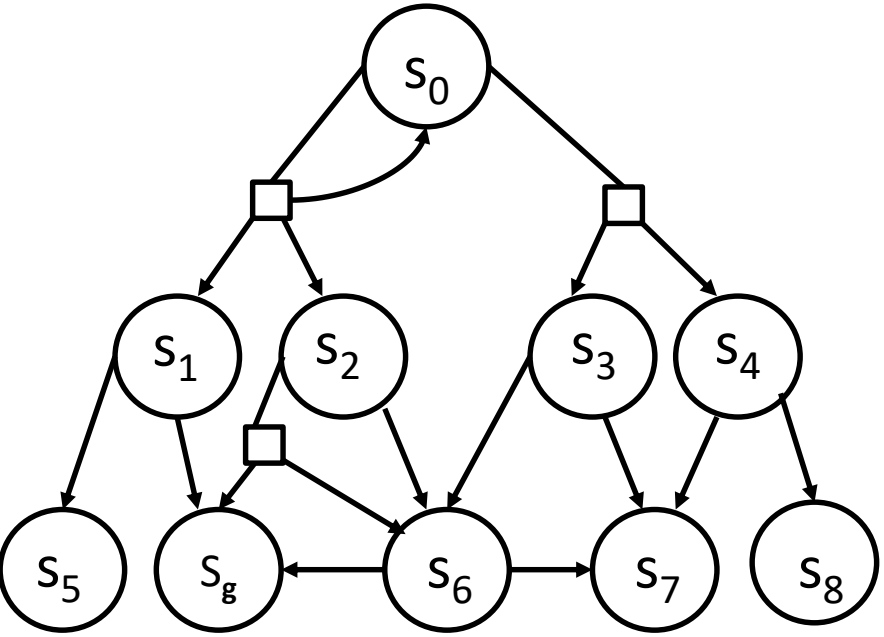
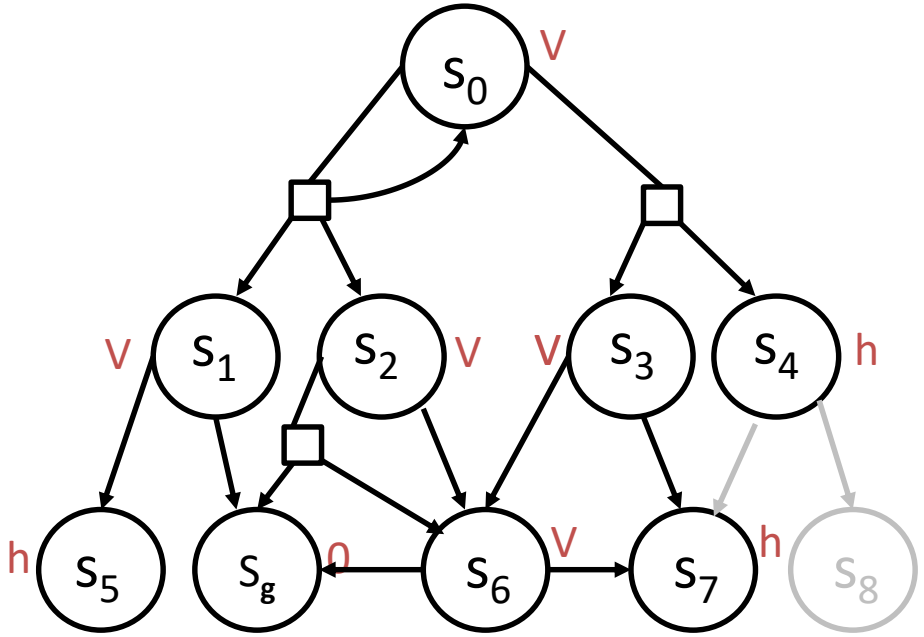




s_4 was never expanded

s_8 was never touched

LAO* [Hansen&Zilberstein 98]

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand best state *s* on the fringe (in greedy graph)
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- perform VI on this subset
- recompute the greedy graph

until greedy graph has no fringe

output the greedy graph as the final policy

one expansion

lot of computation

Optimizations in LAO*

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand best state *s* on the fringe (in greedy graph)
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- VI iterations until greedy graph changes (or low residuals)
- recompute the greedy graph

until greedy graph has no fringe

output the greedy graph as the final policy

Optimizations in LAO*

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand all states in greedy fringe
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- VI iterations until greedy graph changes (or low residuals)
- recompute the greedy graph

until greedy graph has no fringe

output the greedy graph as the final policy

iLAO* [Hansen&Zilberstein 01]

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand all states in greedy fringe
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- only one backup per state in greedy graph
- recompute the greedy graph

until greedy graph has no fringe

in what order?
(fringe → start)
DFS postorder

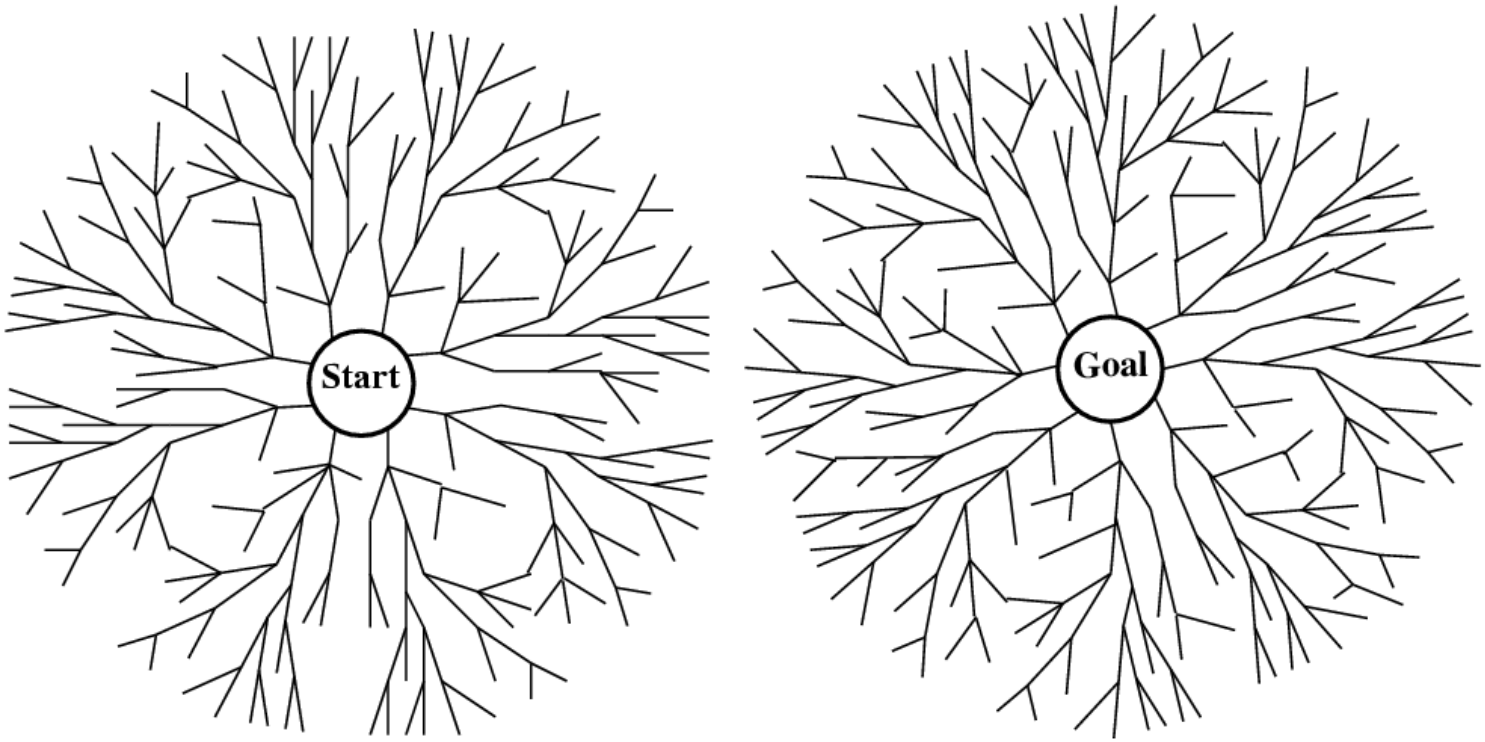
output the greedy graph as the final policy

Reverse LAO* [Dai&Goldsmith 06]

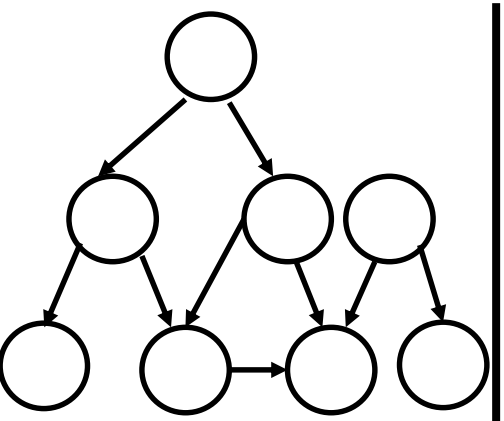
- LAO* may spend huge time until a goal is found
  - guided only by s_0 and heuristic
- LAO* in the reverse graph
  - guided only by goal and heuristic
- Properties
  - Works when 1 or handful of goal states
  - May help in domains with small fan in

Bidirectional LAO* [Dai&Goldsmith 06]

- Go in both directions from start state and goal
- Stop when a bridge is found



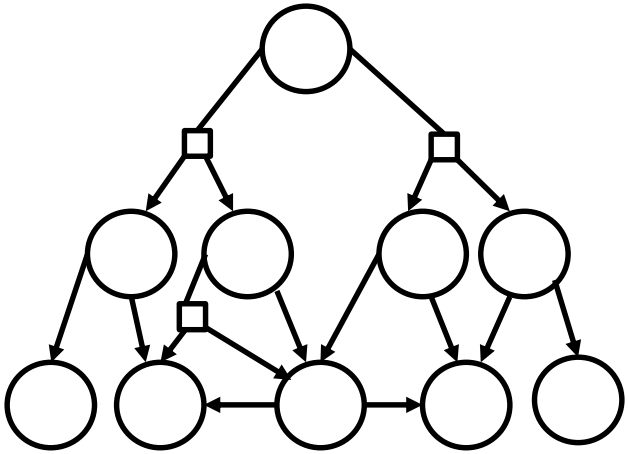
A* → LAO*



regular graph

soln:(shortest) path

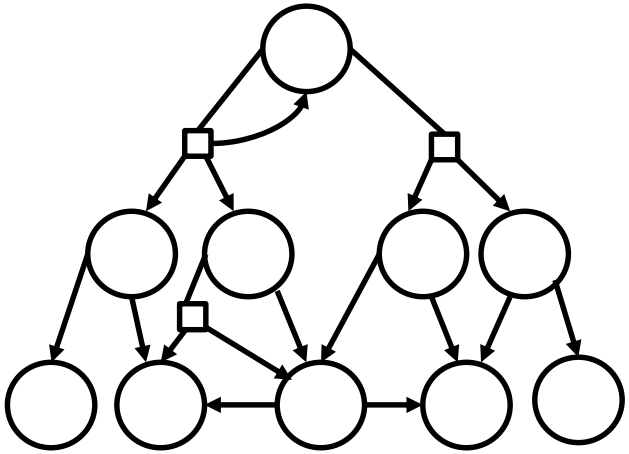
A*



acyclic AND/OR graph

soln:(expected shortest) acyclic graph

AO* [Nilsson'71]



cyclic AND/OR graph

soln:(expected shortest) cyclic graph

LAO* [Hansen&Zil.'98]

All algorithms able to make effective use of reachability information!

AO* for Acyclic MDPs [Nilsson 71]

add s_0 to the fringe and to greedy policy graph

repeat

- FIND: expand best state *s* on the fringe (in greedy graph)
- initialize all new states by their heuristic value
- subset = all states in expanded graph that can reach s
- a single backup pass from fringe states to start state
- recompute the greedy graph

until greedy graph has no fringe

output the greedy graph as the final policy

Heuristic Search Algorithms

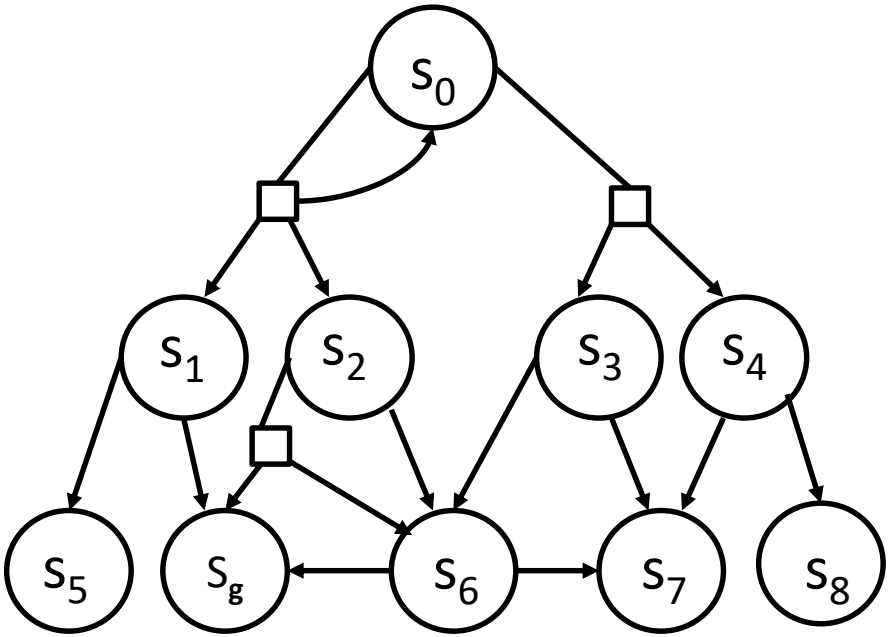
- Definitions
- Find & Revise Scheme.
- LAO* and Extensions
- RTDP and Extensions
- Other uses of Heuristics/Bounds
- Heuristic Design

Real Time Dynamic Programming

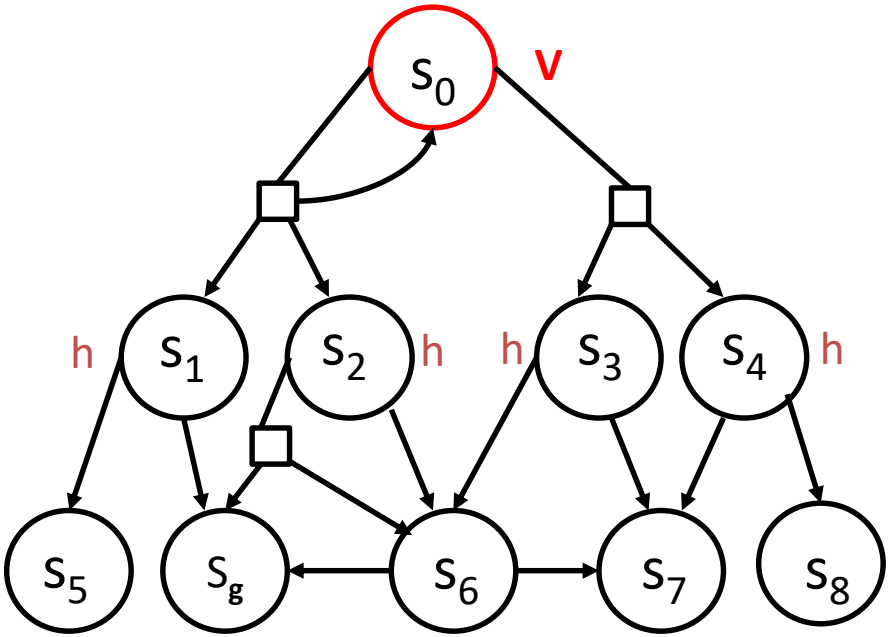
[Barto et al 95]

- Original Motivation
  - agent acting in the real world
- Trial
  - simulate greedy policy starting from start state;
  - perform Bellman backup on visited states
  - stop when you hit the goal
- RTDP: repeat trials forever
  - Converges in the limit #trials $\rightarrow \infty$

Trial



Trial



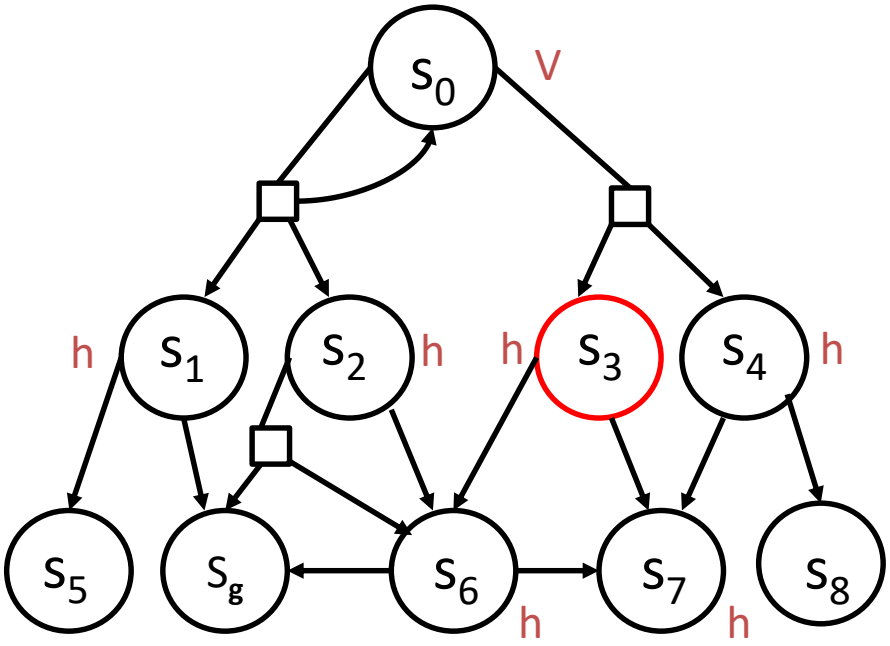
start at start state

repeat

perform a Bellman backup

simulate greedy action

Trial



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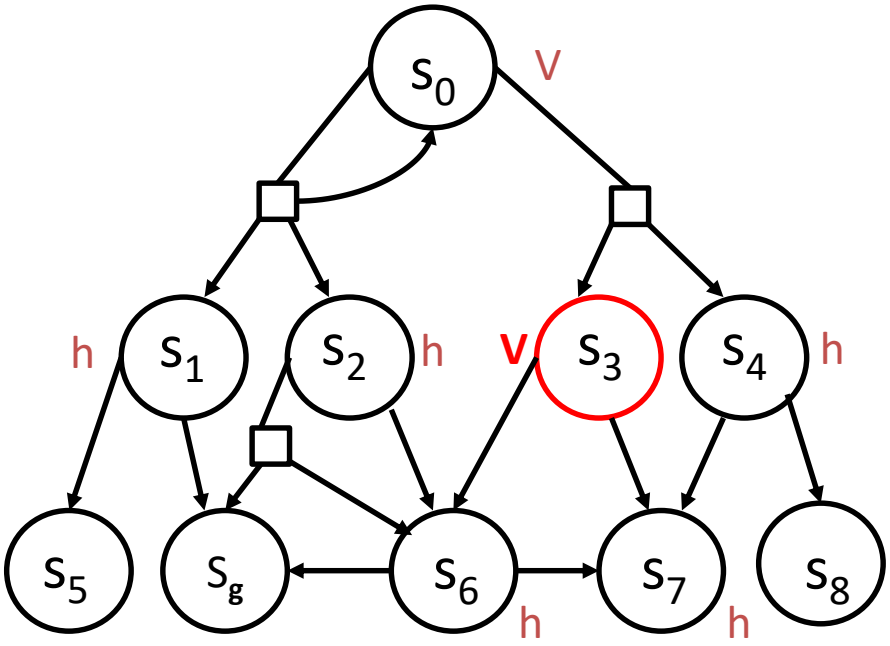


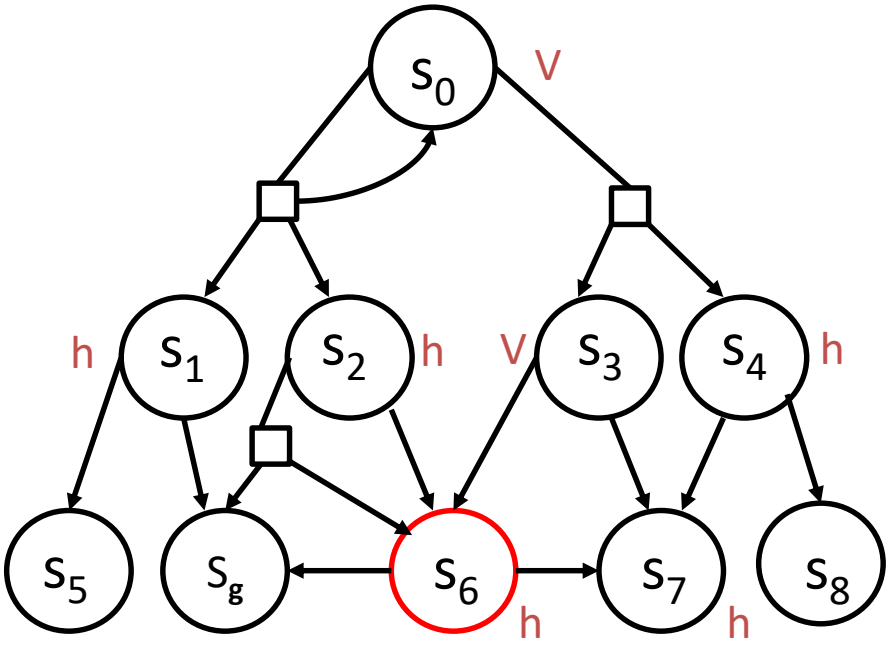
start at start state

repeat

perform a Bellman backup

simulate greedy action

Trial



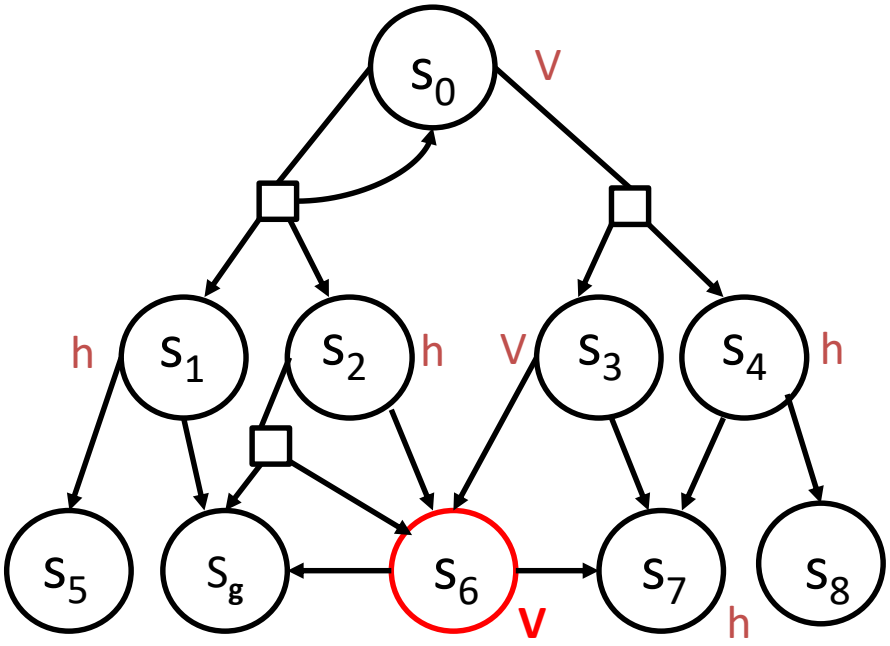
start at start state

repeat

perform a Bellman backup

simulate greedy action

Trial



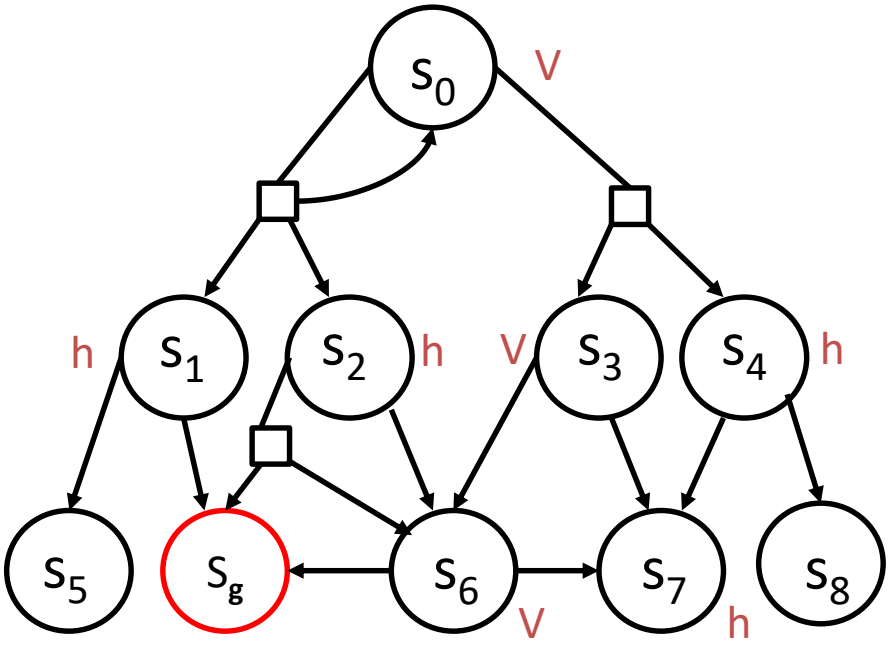
start at start state

repeat

perform a Bellman backup

simulate greedy action

Trial



start at start state

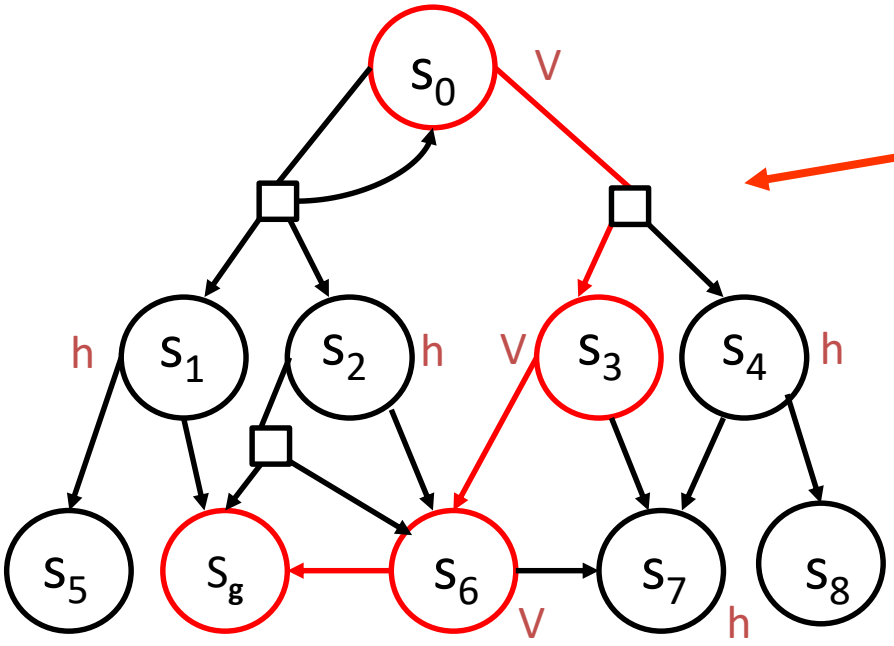
repeat

perform a Bellman backup

simulate greedy action

until hit the goal

Trial



RTDP

repeat forever

start at start state
repeat
    perform a Bellman backup
    simulate greedy action
until hit the goal

Real Time Dynamic Programming

[Barto et al 95]

- Original Motivation
  - agent acting in the real world
- Trial
  - simulate greedy policy starting from start state;
  - perform Bellman backup on visited states
  - stop when you hit the goal
- RTDP: repeat trials forever
  - Converges in the limit #trials $\rightarrow \infty$

No termination condition!

RTDP Family of Algorithms

repeat

    $s \leftarrow s_0$

    repeat *//trials*

        REVISE s; identify a_{greedy}

        FIND: pick s' s.t. $T(s, a_{greedy}, s') > 0$

        $s \leftarrow s'$

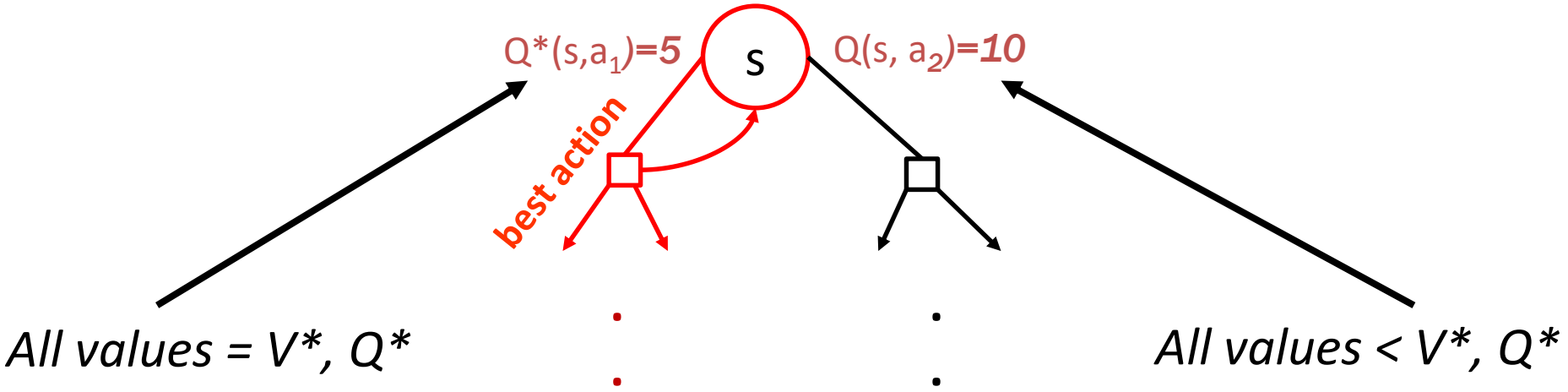
    until $s \in G$

until termination test

F&R and Monotonicity

- $V_k \leq_p V^* \Rightarrow V_n \leq_p V^*$ (V_n monotonic from below)
  - If h is admissible: $V_0 = h(s) \leq_p V^*$

  $\Rightarrow V_n \leq_p V^*$ ($\forall n$)



$Q^*(s,a_1) < Q(s,a_2) < Q^*(s,a_2)$

a_2 can't be optimal

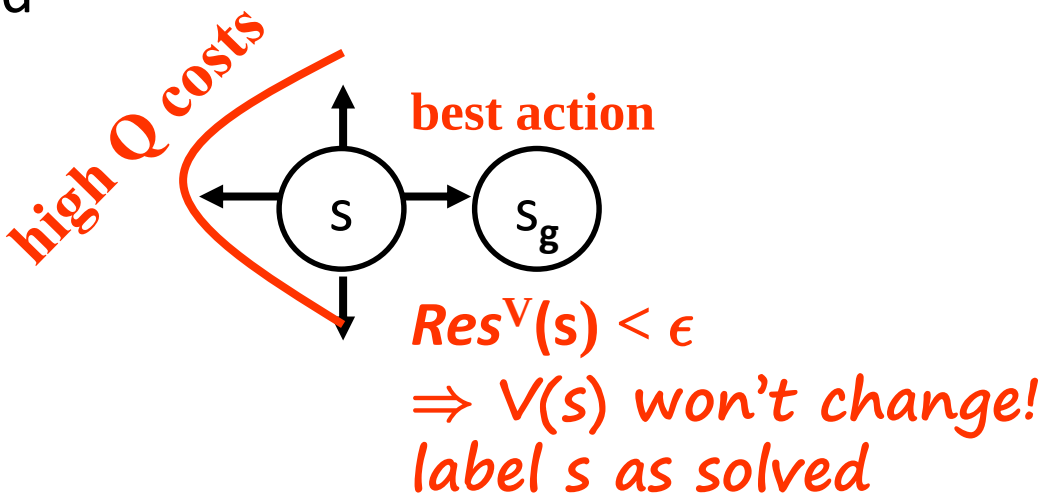
Termination Test Take 1: Labeling

- Admissible heuristic & monotonicity

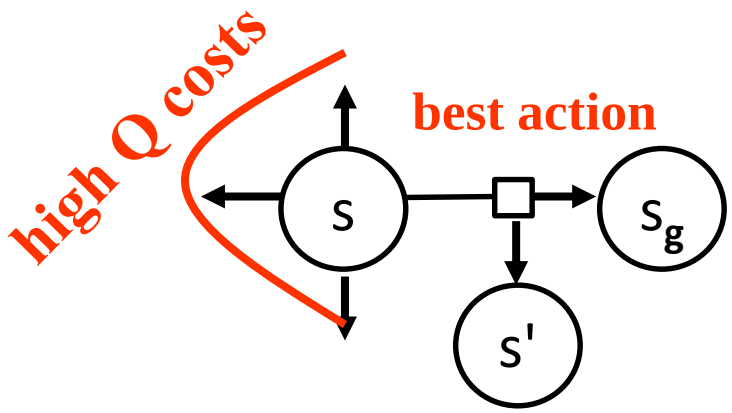
  $\Rightarrow V(s) \leq V^*(s)$

  $\Rightarrow Q(s,a) \leq Q^*(s,a)$

- Label a state s as solved
  - if V(s) has converged



Labeling (contd)



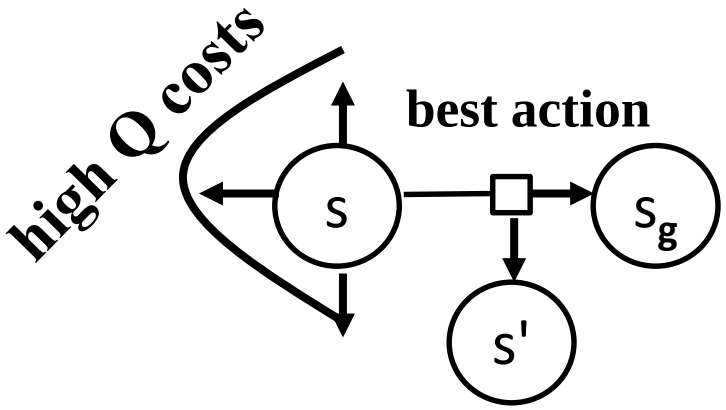
$Res^V(s) < \epsilon$

s' already solved

$\Rightarrow$ V(s) won't change!

label s as solved

Labeling (contd)

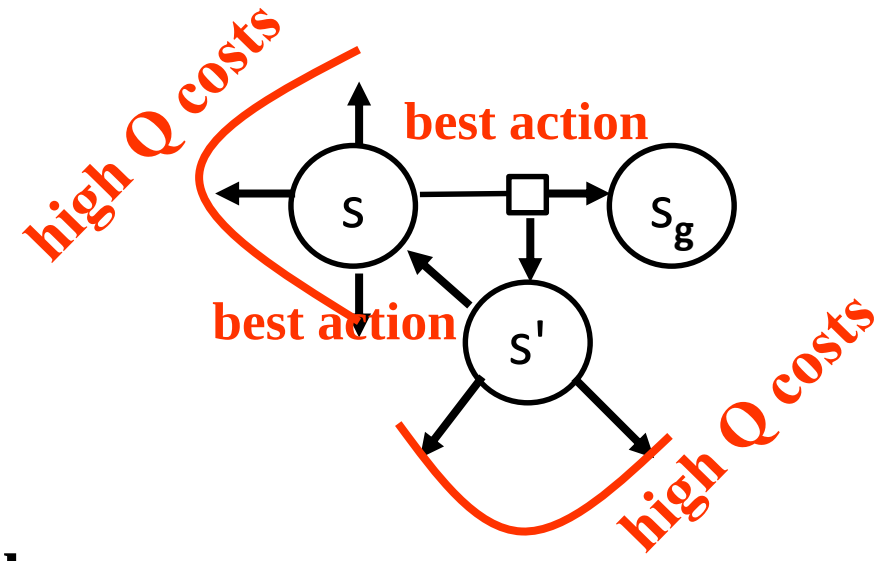


$\text{Res}^V(s) < \epsilon$

s' already solved

$\Rightarrow$ V(s) won't change!

label s as solved



$\text{Res}^V(s) < \epsilon$

$\text{Res}^V(s') < \epsilon$

V(s), V(s') won't change!
label s, s' as solved

Labeled RTDP [Bonet&Geffner 03b]

repeat

    $s \leftarrow s_0$

    label all goal states as solved

    repeat *//trials*

        REVISE s; identify a_{greedy}

        FIND: sample s' from $T(s, a_{greedy}, s')$

        $s \leftarrow s'$

    until s is solved

    for all states s in the trial

        try to label s as solved

until s_0 is solved

LRTDP

- terminates in finite time
  - due to labeling procedure
- anytime
  - focuses attention on more probable states
- fast convergence
  - focuses attention on unconverged states

Picking a Successor Take 2

- Labeled RTDP/RTDP: sample $s' \propto T(s, a_{greedy}, s')$
  - Adv: more probable states are explored first
  - Labeling Adv: no time wasted on converged states
  - Disadv: labeling is a hard constraint
  - Disadv: sampling ignores "amount" of convergence

- If we knew how much V(s) is expected to change?
  - sample $s' \propto$ expected change

Upper Bounds in SSPs

- RTDP/LAO* maintain lower bounds
  - call it V_l
- Additionally associate upper bound with s
  - $V_u(s) \geq V^*(s)$
- Define $gap(s) = V_u(s) - V_l(s)$
  - low gap(s): more converged a state
  - high gap(s): more expected change in its value

Backups on Bounds

- Recall monotonicity
- Backups on lower bound
  - continue to be lower bounds
- Backups on upper bound
  - continues to be upper bounds
- Intuitively
  - V_l will increase to converge to V*
  - V_u will decrease to converge to V*

Bounded RTDP [McMahan et al 05]

repeat

$s \leftarrow s_0$

repeat *//trials*

identify a_{greedy} based on V_l

FIND: sample $s' \propto T(s, a_{greedy}, s').gap(s')$

$s \leftarrow s'$

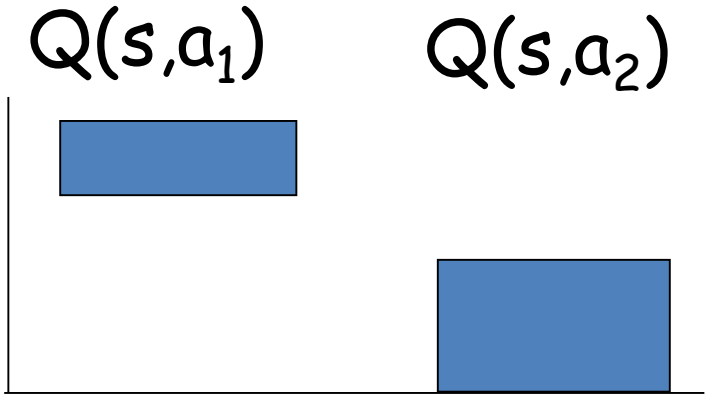
until $gap(s) < \epsilon$

for all states s in trial in reverse order

REVISE s

until $gap(s_0) < \epsilon$

Action Elimination



If $Q_l(s,a_1) > Q_u(s,a_2)$ then a_1 cannot be optimal for s.

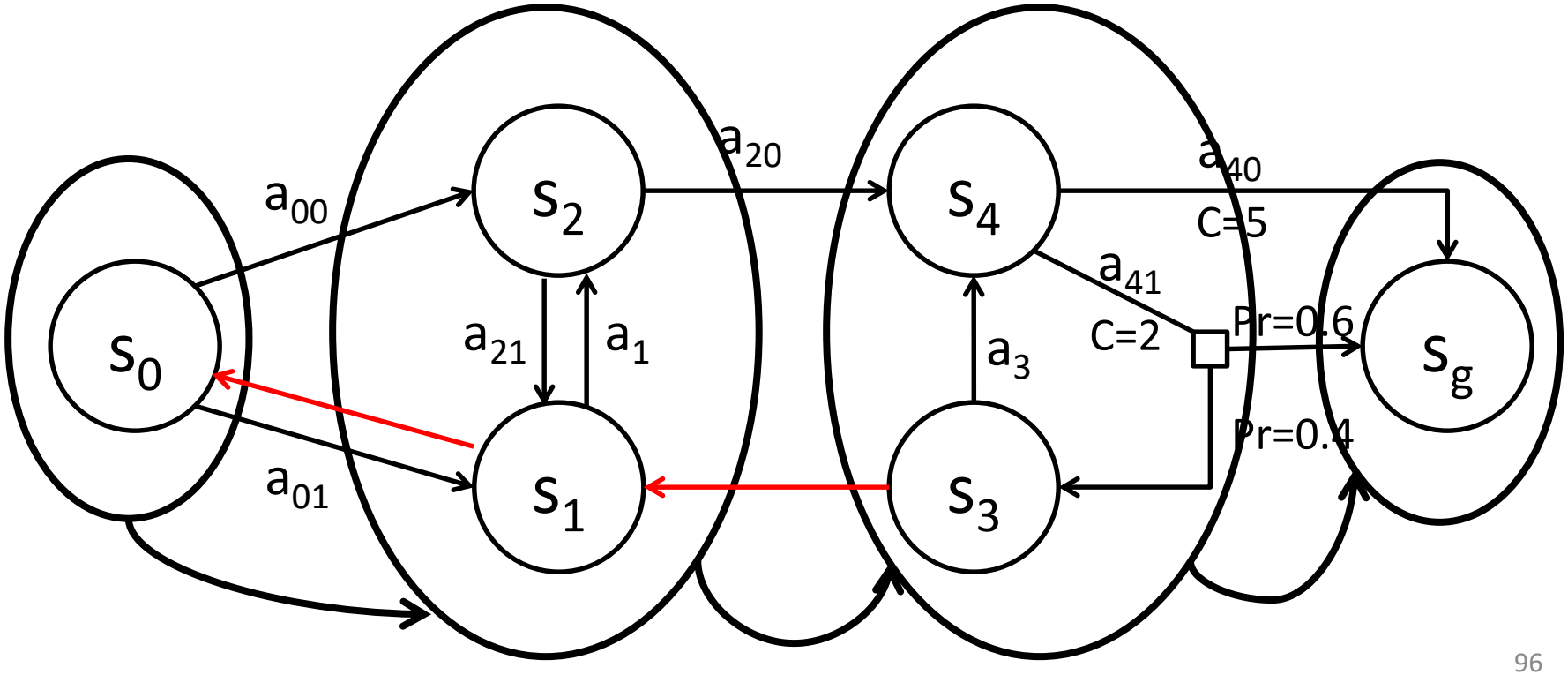
Leads to VPI-RTDP [Sanner et al 09]

Heuristic Search Algorithms

- Definitions
- Find & Revise Scheme.
- LAO* and Extensions
- RTDP and Extensions
- Other uses of Heuristics/Bounds
- Heuristic Design

Topological VI [Dai&Goldsmith 07]

- Identify strongly-connected components
- Perform topological sort of partitions
- Backup partitions to ϵ-consistency: reverse top. order



Topological VI [Dai&Goldsmith 07]

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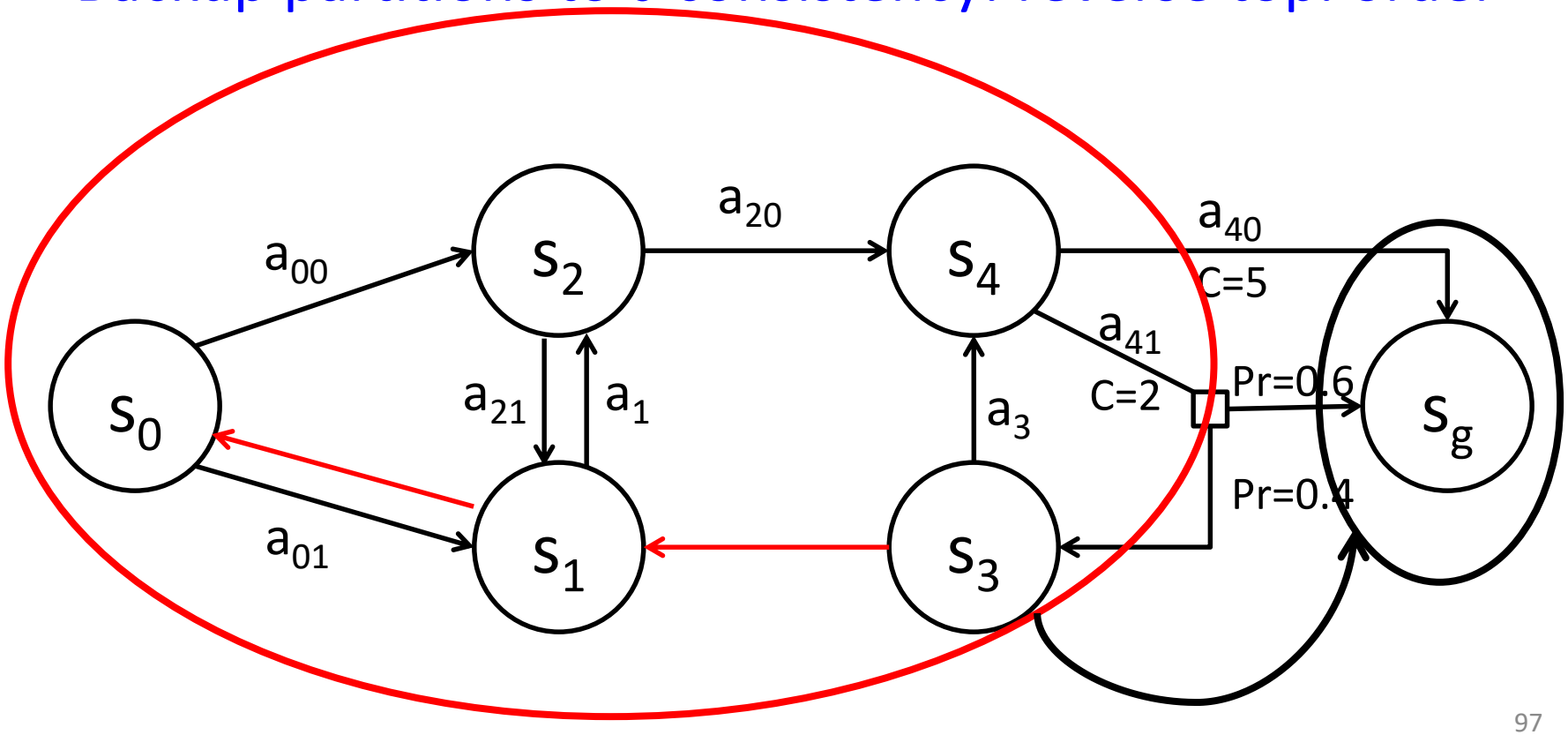


Focused Topological VI [Dai, Mausam, Weld 09]

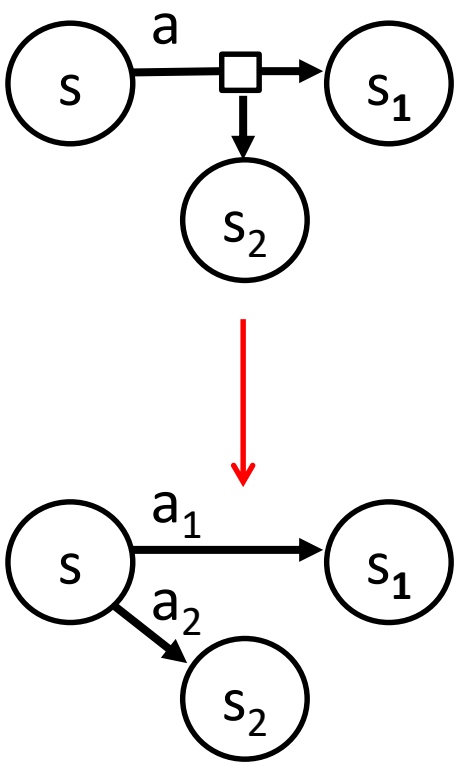
- Topological VI
  - hopes there are many small connected components
  - can‘t handle reversible domains…
- FTVI
  - initializes V_l and V_u
  - LAO*-style iterations to update V_l and V_u
  - eliminates actions using action-elimination
  - Runs TVI on the resulting graph

Heuristic Search Algorithms

- Definitions
- Find & Revise Scheme.
- LAO* and Extensions
- RTDP and Extensions
- Other uses of Heuristics/Bounds
- Heuristic Design

Admissible Heuristics

- Basic idea
  - Relax probabilistic domain to deterministic domain
  - Use heuristics(classical planning)

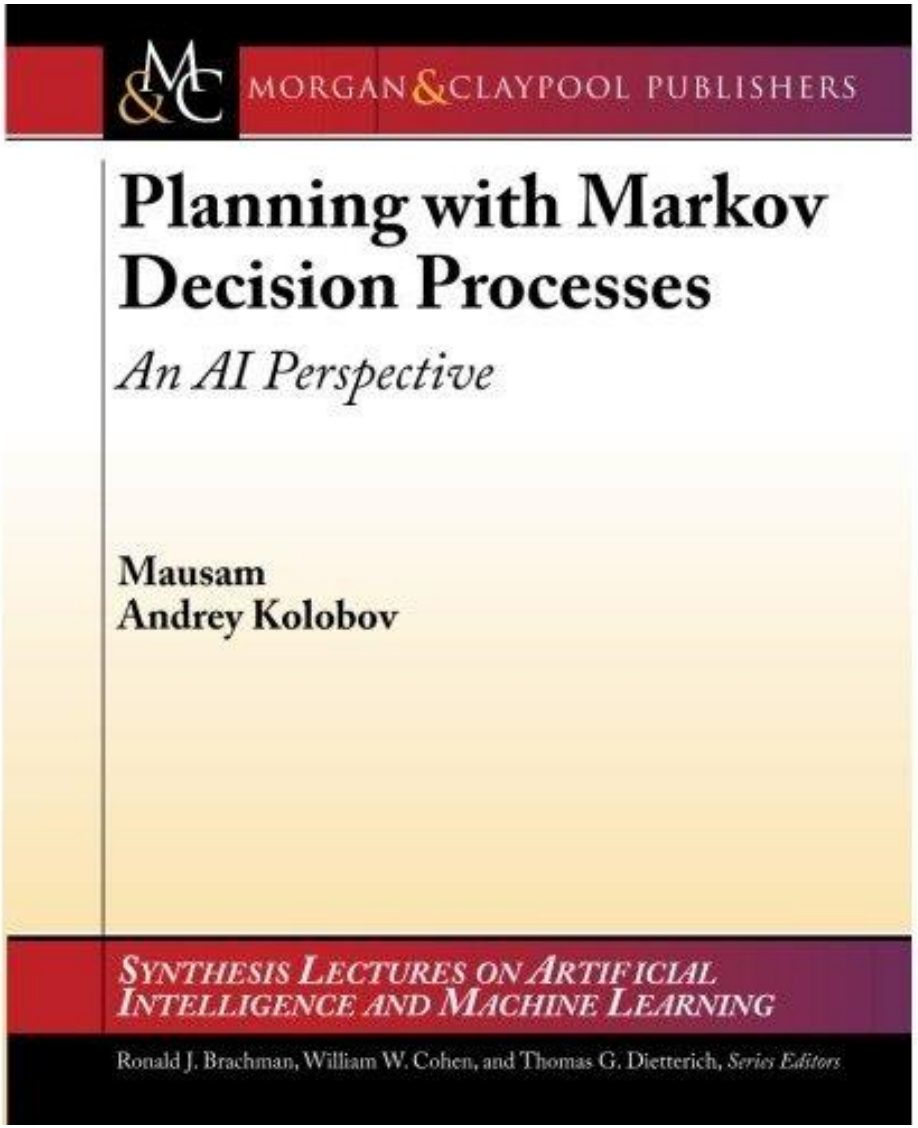


- All-outcome Determinization
  - For each outcome create a different action
- Admissible Heuristics
  - Cheapest cost solution for determinized domain
  - Classical heuristics over determinized domain

Summary of Heuristic Search

- Definitions
- Find & Revise Scheme
  - General scheme for heuristic search
- LAO* and Extensions
  - LAO*, iLAO*, RLAO*, BLAO*
- RTDP and Extensions
  - RTDP, LRTDP, BRTDP, FRTDP, VPI-RTDP
- Other uses of Heuristics/Bounds
  - Action Elimination, FTVI
- Heuristic Design
  - Determinization-based heuristics

Shameless Plug



Determinization-based Algorithms for SSPs and Beyond

Mausam

Computer Science and Engineering

Indian Institute of Technology (IIT) Delhi

PPDDL

- PDDL/STRIPS
  - precondition: conjunction of fluents
  - effect: all changes in the state
- PPDDL
  - precondition: as above
  - LIST of (effect, probability)
- RDDL
  - concurrent effects, natural dynamics

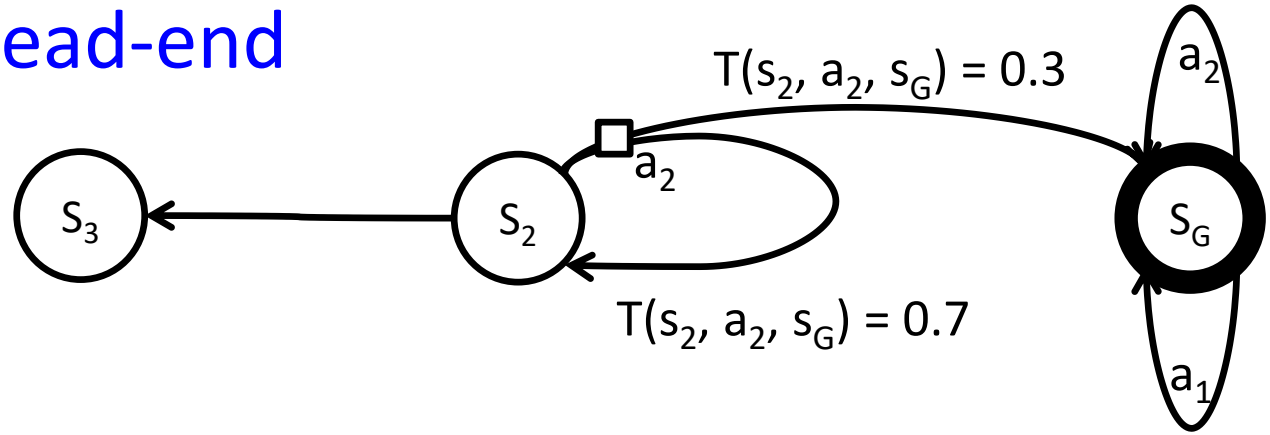
BEYOND SSPs

Domains with Dead-ends

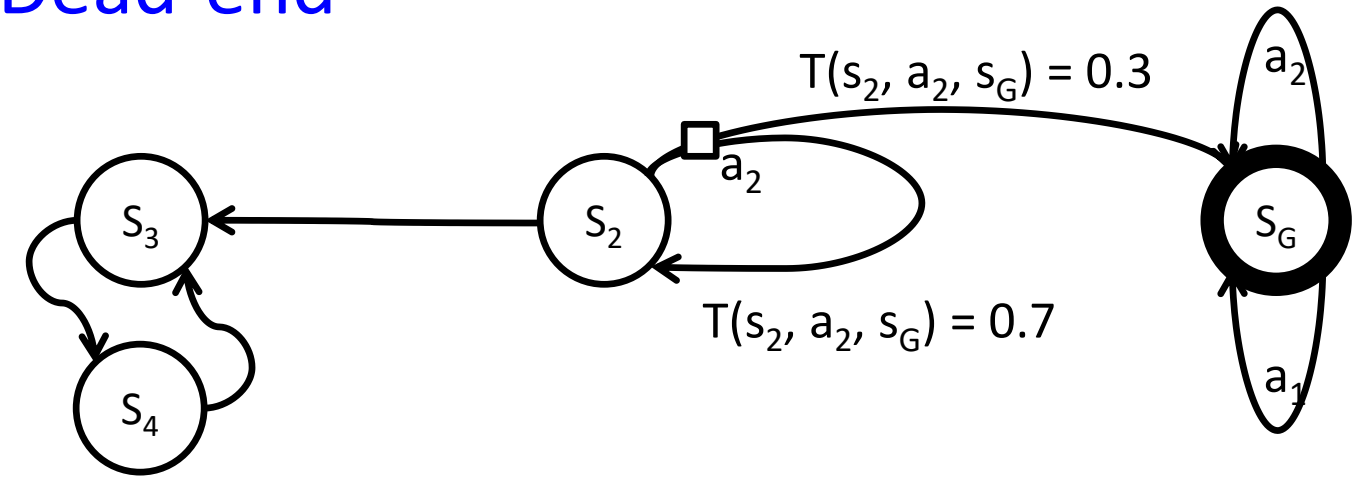
- Dead-end state
  - a state from which goal is unreachable
- Common in real-world
  - rover
  - traffic
  - exploding blocksworld!
- SSP/SSP_{s0} do not model such domains
  - assumption of “at-least one proper policy” violated

Two Types of Dead-ends

- Explicit Dead-end



- Implicit Dead-end



Modeling Dead-ends

- How should we model dead-end states?
  - V(s) is undefined for deadends

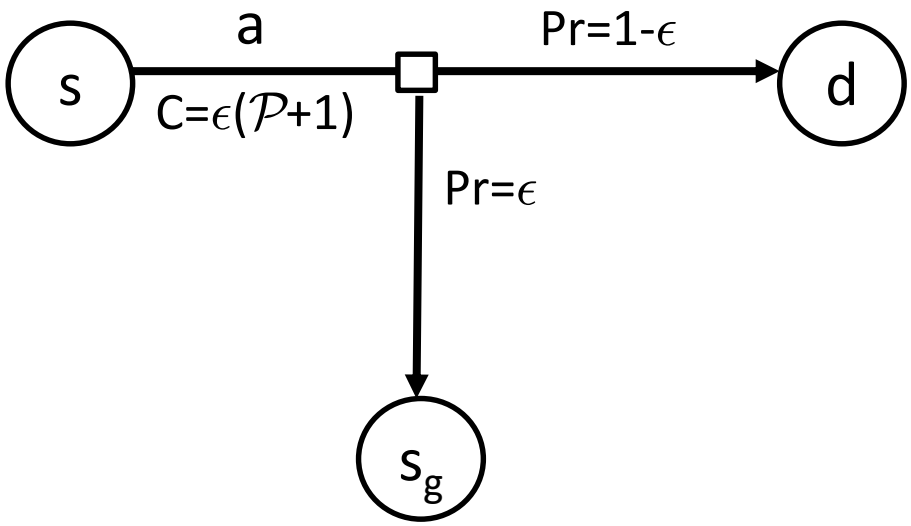
  ⇒ VI does not converge!!

- Proposal 1
  - Add a penalty of reaching the dead-end state = $\mathcal{P}$

- Is everything well-formed?

- Are there any issues with the model?

Simple Dead-end Penalty $\mathcal{P}$



- $V^*(s) = \epsilon(\mathcal{P}+1) + \epsilon.0 + (1-\epsilon).\mathcal{P}$
  $= \mathcal{P} + \epsilon$

V(non-deadend) > $\mathcal{P}$

Proposal 2

- fSSPDE: Finite-Penalty SSP with Deadends
- Agent allowed to stop at ***any*** state
  - by paying a price = penalty $\mathcal{P}$
  - Intuition: achieving a goal is worth $-\mathcal{P}$ to the agent

$$V^*(s) = \min\left(\mathcal{P}, \min_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{T}(s,a,s')\mathcal{C}(s,a,s') + V^*(s')]\right)$$

- Equivalent to SSP with special a_{stop} action
  - applicable in each state
  - leads directly to goal by paying cost $\mathcal{P}$
- SSP = fSSPDE

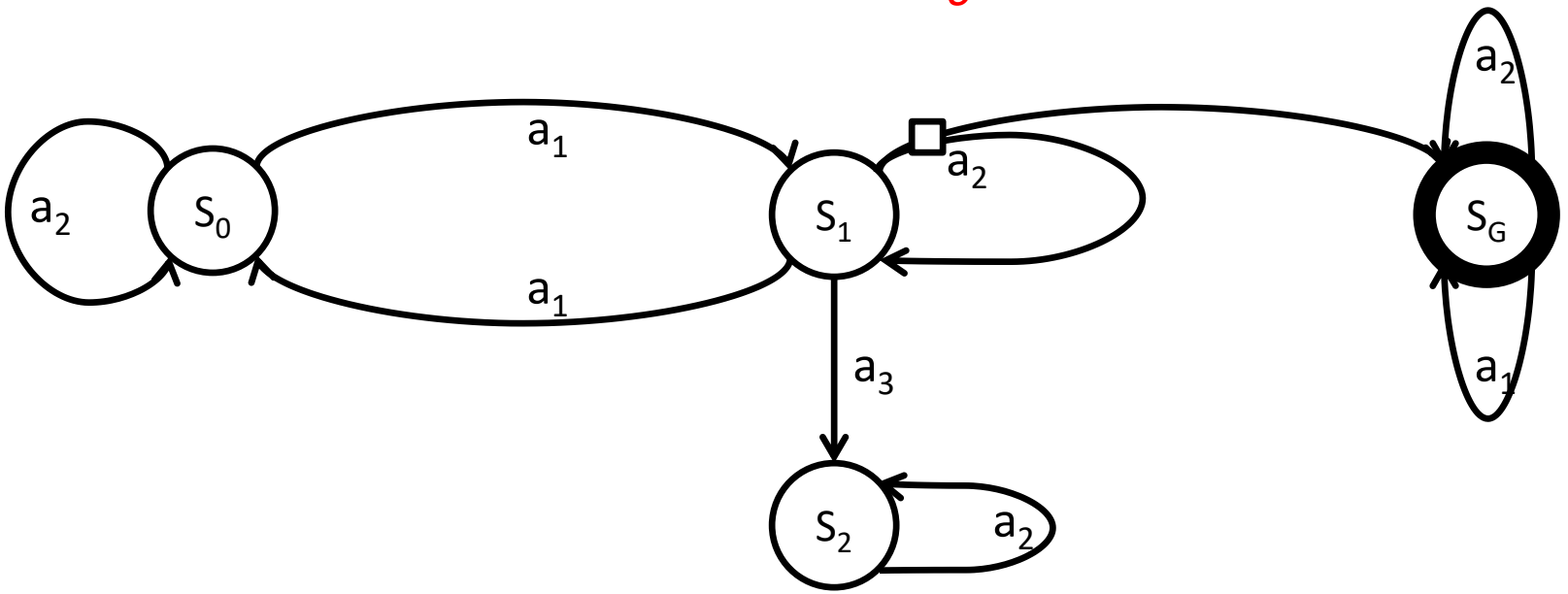
fSSPDE Algorithms

- All SSP algorithms applicable…
- Efficiency: unknown so far…
  - Efficiency hit due to presence of deadends
  - Efficiency hit due to magnitude of $\mathcal{P}$
  - Efficiency hit due to change of topology (e.g., TVI)

SSPADE: Dead Ends are Avoidable from s_0

[Kolobov, Mausam, Weld, UAI'12]

- D.e.s may be avoidable *from* s_0 via an optimal policy



- Can't compute *V*(s)* for every state
- But need only "relevant" states to get the "right" value
- there exists a proper (partial) policy rooted at s_0

SSPADE

- Can be solved with optimal heuristic search from s_0
  - FIND shouldn't starve states; REVISE should halt
- Heuristic Search Algorithms
  - LAO*: may not converge
    - V(dead-ends) will get unbounded: VI may not converge
  - iLAO*: will converge
    - only 1 backup $\Rightarrow$ greedy policy will exit dead-ends
  - RTDP/LRTDP: may not converge
    - once stuck in dead-end → won't reach the goal
    - add max #steps in a trial… how many? adaptive?

Unavoidable Dead-ends

- fSSPUDE: Finite-Penalty SSP with Unavoidable Dead-Ends [Kolobov et al 12]
  - same as fSSPDE but now with a start state
- Same transformation applies
  - add an a_{stop} action from every state
- SSP_{s0} = fSSPUDE

Unavoidable Dead-ends

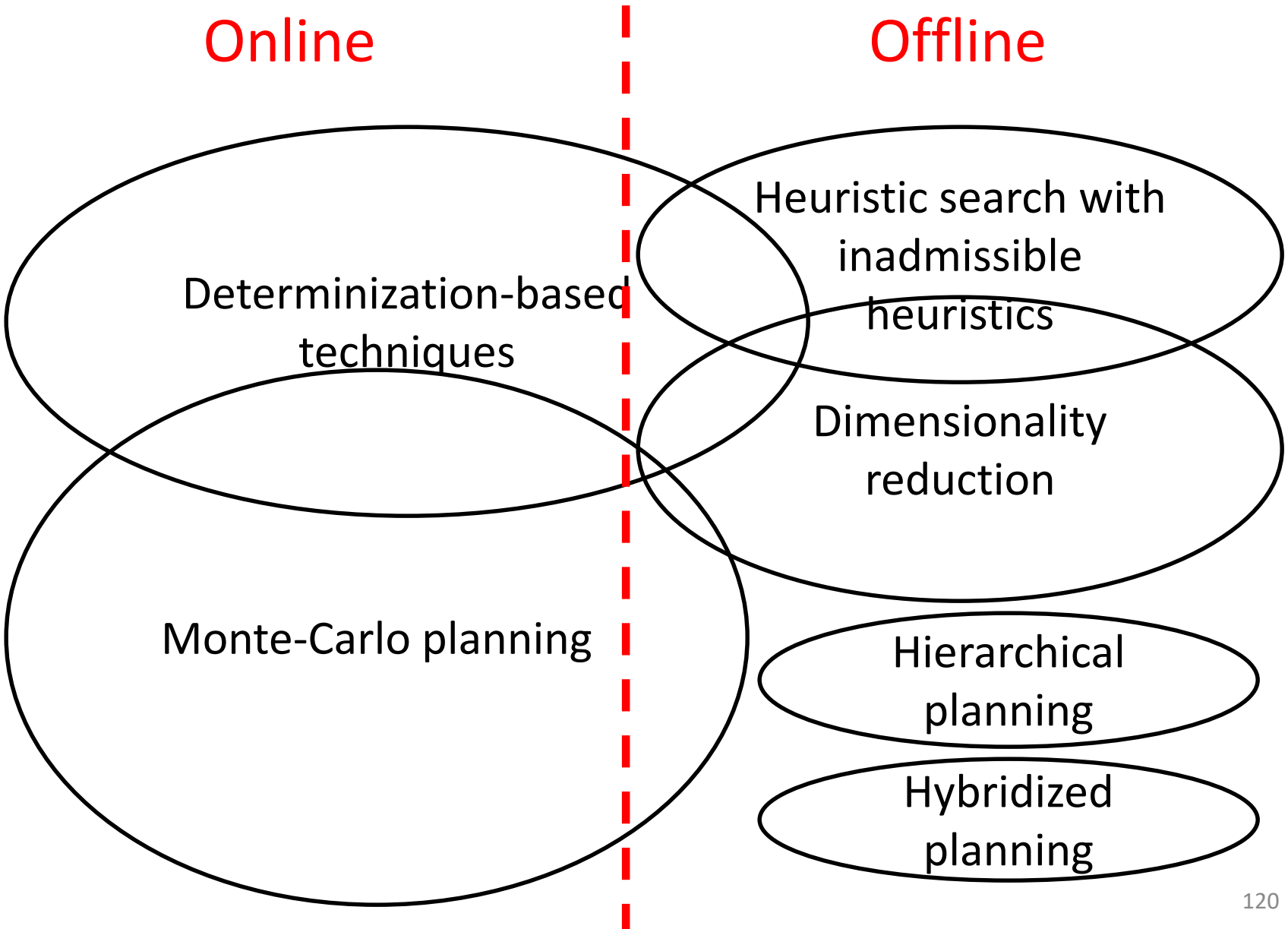
- iSSPUDE: Infinite-Penalty SSP with Unavoidable Dead-Ends [Kolobov et al 12]
  - (MAXPROB) find policy that first maximizes the prob of reaching goal
  - from all such policies find one minimum expected cost
- Dual objective
- iSSPUDE is much harder than SSP_{s0}

DETERMINIZATION-BASED APPROXIMATION ALGORITHMS

Motivation

- Even π^* closed wr.t. s_0 is often too large to fit in memory…
- … and/or too slow to compute …
- … for MDPs with complicated characteristics
  - Large branching factors/high-entropy transition function
  - Large distance to goal
  - Etc.
- **Must sacrifice optimality to get a "good enough" solution**

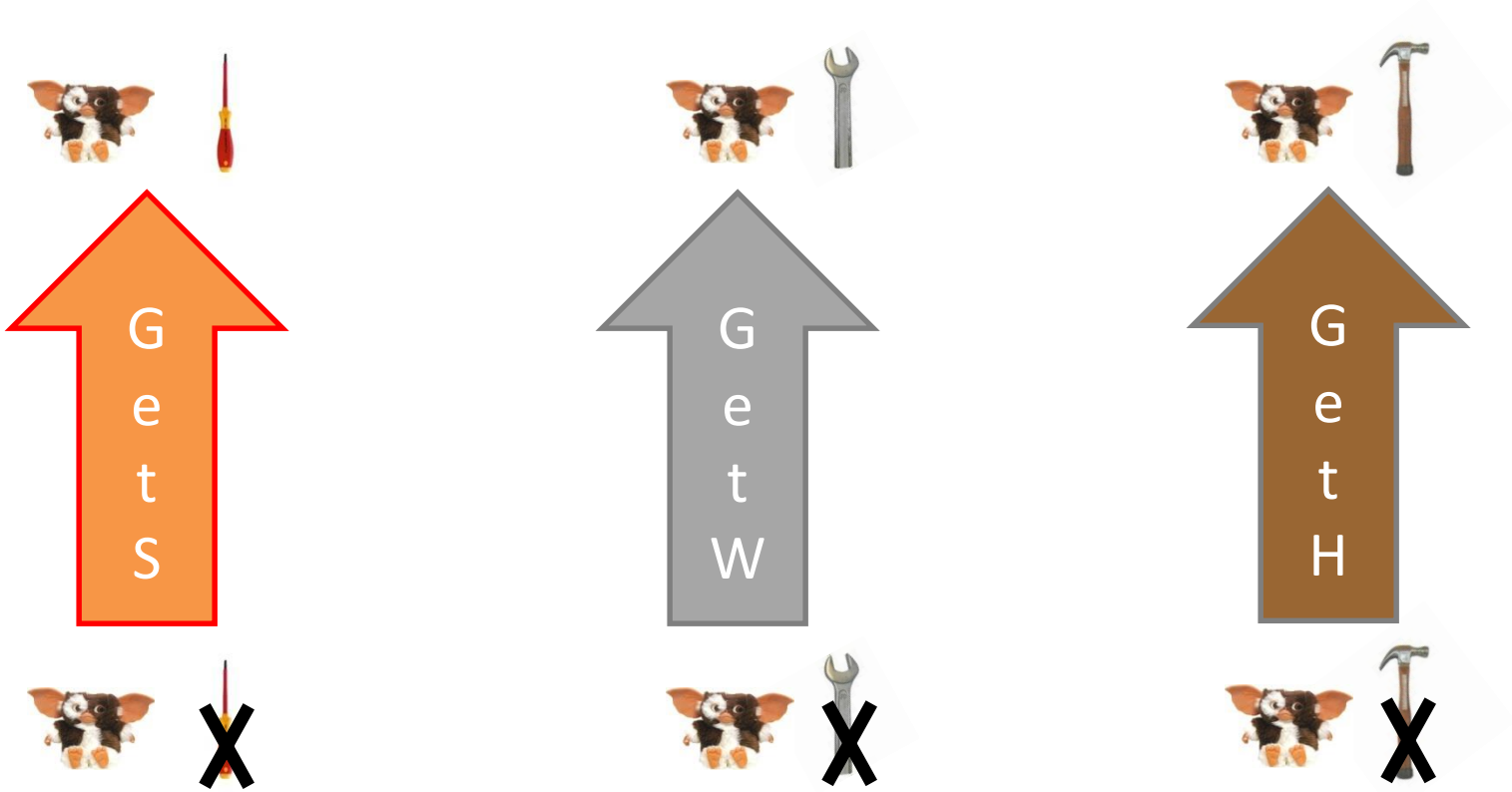
Approximation Ideas



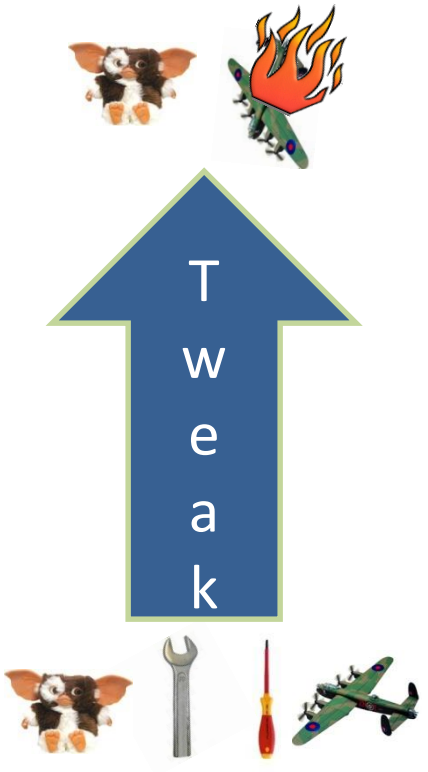
Overview

- Not a “golden standard” classification
  - Others possible, e.g., optimal in the limit vs. suboptimal in the limit
- Most techniques assume factored fSSPUDE MDPs (SSP_{s0} MDPs with a finite dead-end penalty)
- Approaches differ in the quality aspect they sacrifice
  - Probability of reaching the goal
  - Expected cost of reaching the goal
  - Both

Example Domain



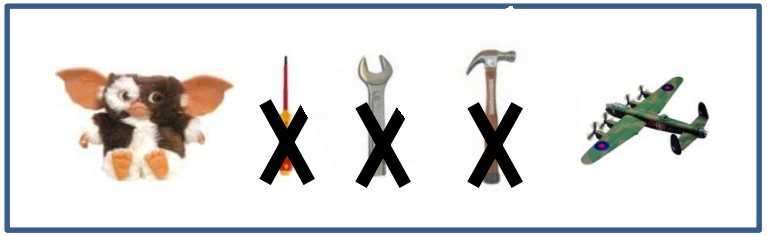
Example Domain (cont'd)



SSP_{s0} MDP

- S: A set of states
- A: A set of actions
- T(s,a,s'): transition model
- C(s,a,s'): action cost
- s_0: start state
- G: set of goals

GetW, GetH, GetS, Tweak, Smash



Outline

- Online Algorithms
  - FF Replan
  - FF Hindsight
  - RFF
- Offline Algorithms
  - Inadmissible Heuristics
  - Dimensionality Reduction
  - Other Determinization Approaches

Online Algorithms: Motivation

- **Defining characteristics:**
  - Planning + execution are interleaved
  - Little time to plan
    - Need to be fast!
  - Worthwhile to compute policy only for visited states
    - Would be wasteful for all states

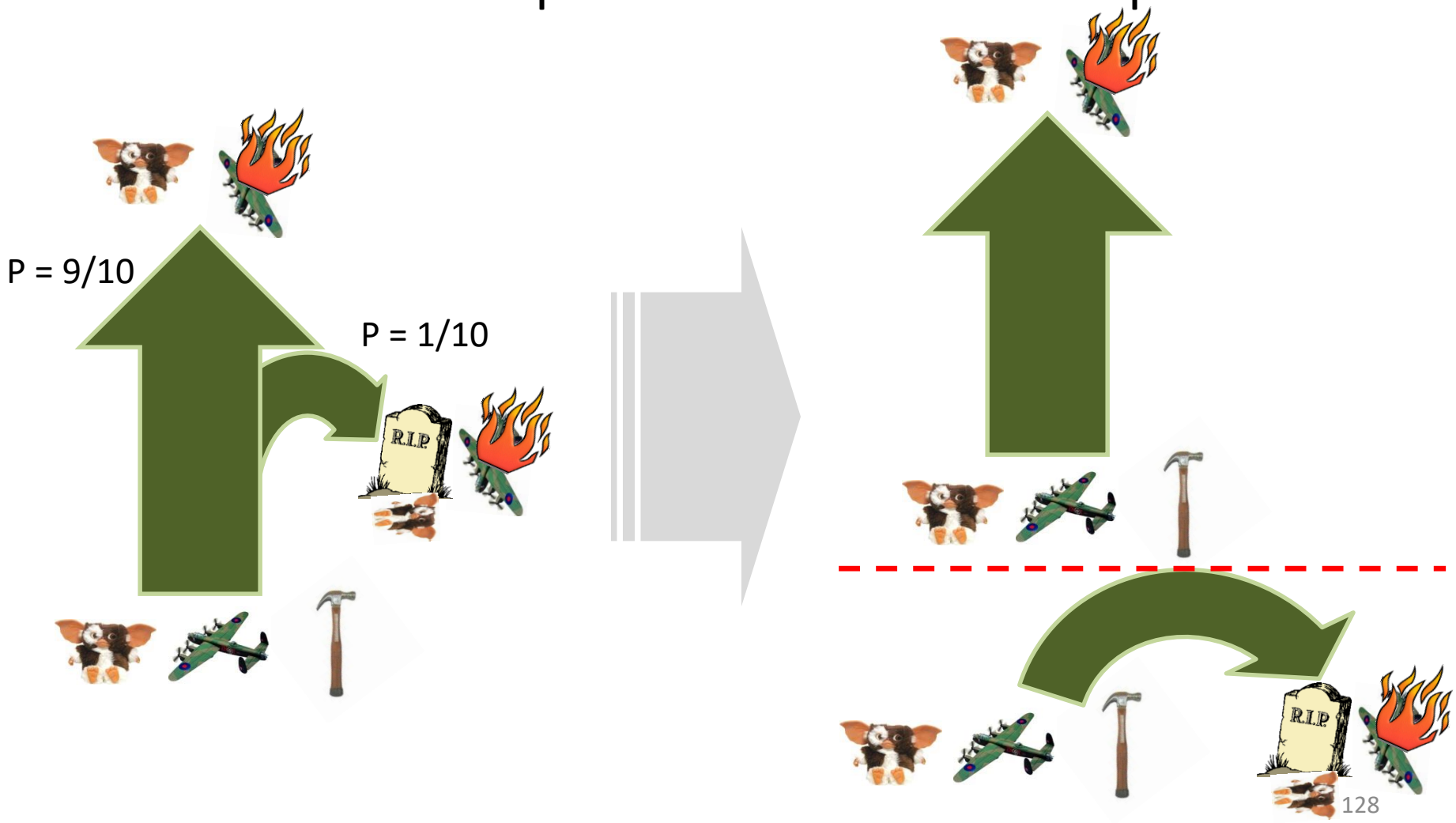
Determinization-based Techniques

- **A way to get a quick'n'dirty solution:**
  - Turn the MDP into a *classical* planning problem
  - Classical planners are comparatively *very* fast

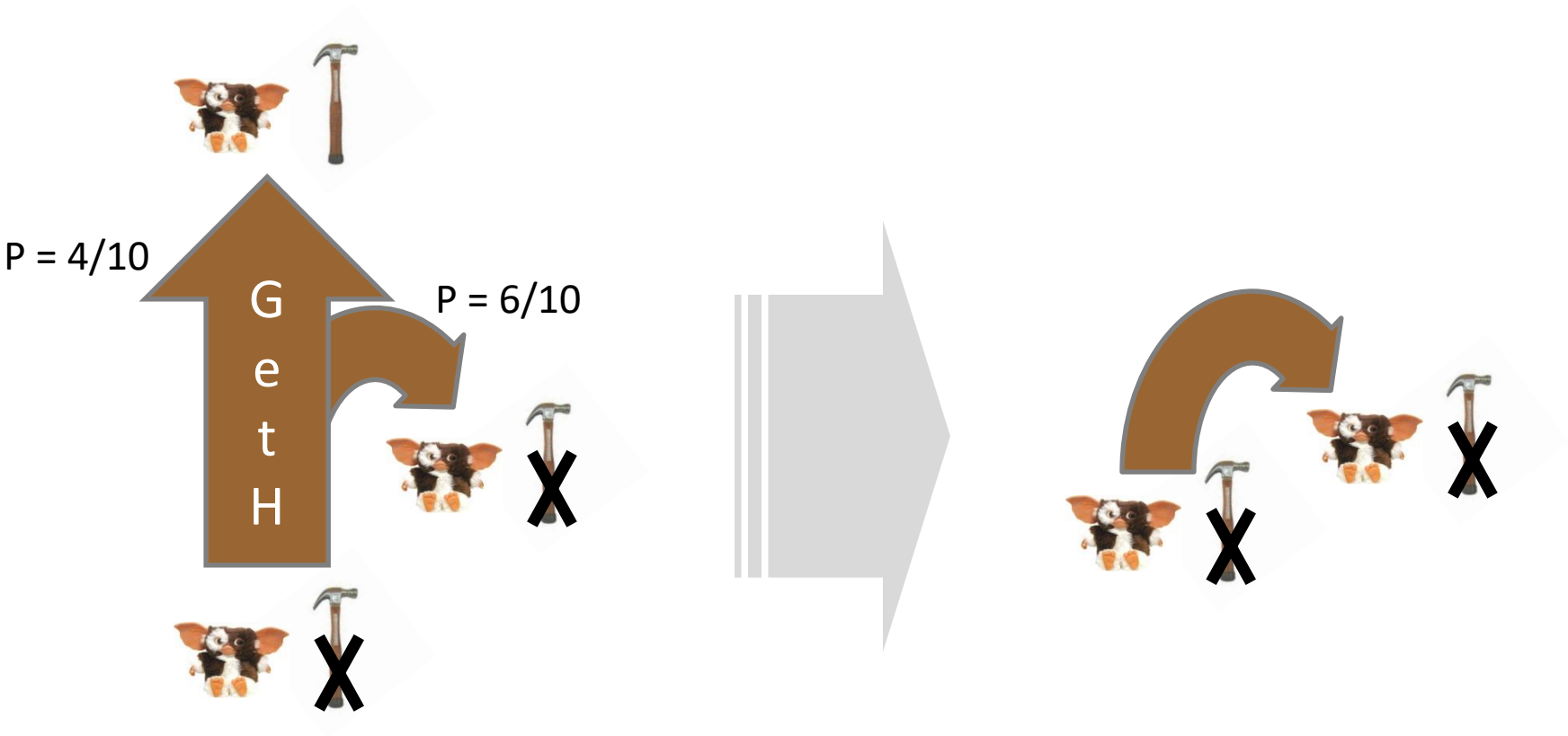
- Main idea:
  1. Compile MDP into its *determinization*
  2. Generate plans in the determinization
  3. Use the plans to choose an action in the curr. state
  4. Execute, repeat

All-Outcome Determinization

Each outcome of each probabilistic action → separate action



Most-Likely-Outcome Determinization



[Yoon, Fern, Givan 2007]

FF-Replan: Overview & Example

1) Find a goal plan in a determinization

2) Try executing it in the original MDP

3) Replan&repeat if unexpected outcome

FF-Replan: Details

- Uses either the AO or the MLO determinization
  - MLO is smaller/easier to solve, but
  - AO contains all possible plans, but
- Uses the *FF* planner to solve the determinization
  - Super fast
  - Other fast planners, e.g., LAMA, possible
- Does not cache computed plans
  - Recomputes the plan in the 3rd step in the example

FF-Replan: Theoretical Properties

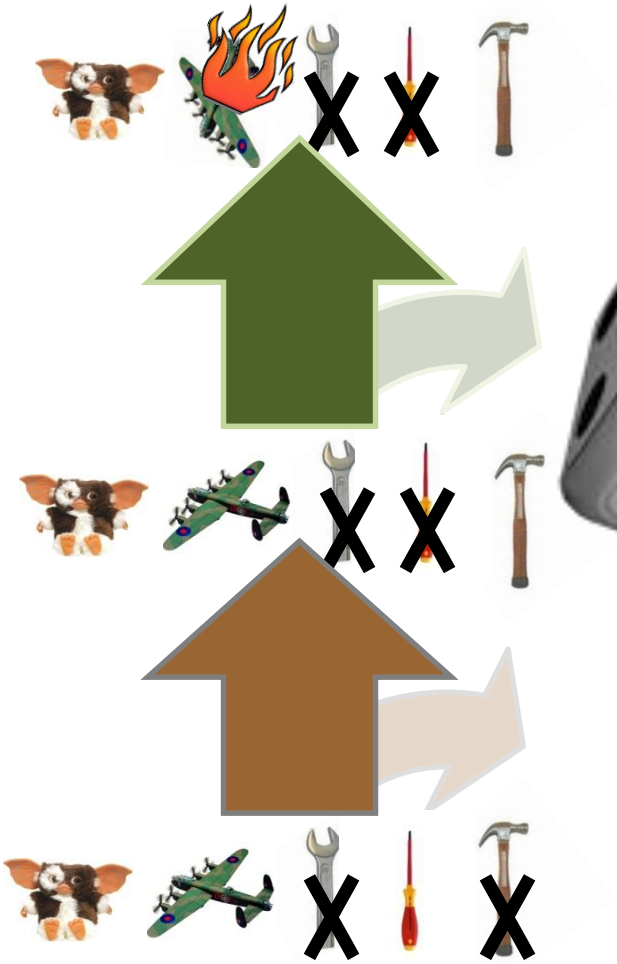
- Optimizes the *MAXPROB* criterion in SSPs
  - In SSPs, this is always 1.0
  - Super-efficient on SSPs w/o dead ends
  - Largely ignores expected cost

- Ignores probability of deviation from the found plan
  - Results in long-winded paths to the goal
  - Troubled by *probabilistically interesting MDPs* [Little, Thiebaux, 2007]
    - There, an unexpected outcome may lead to catastrophic consequences

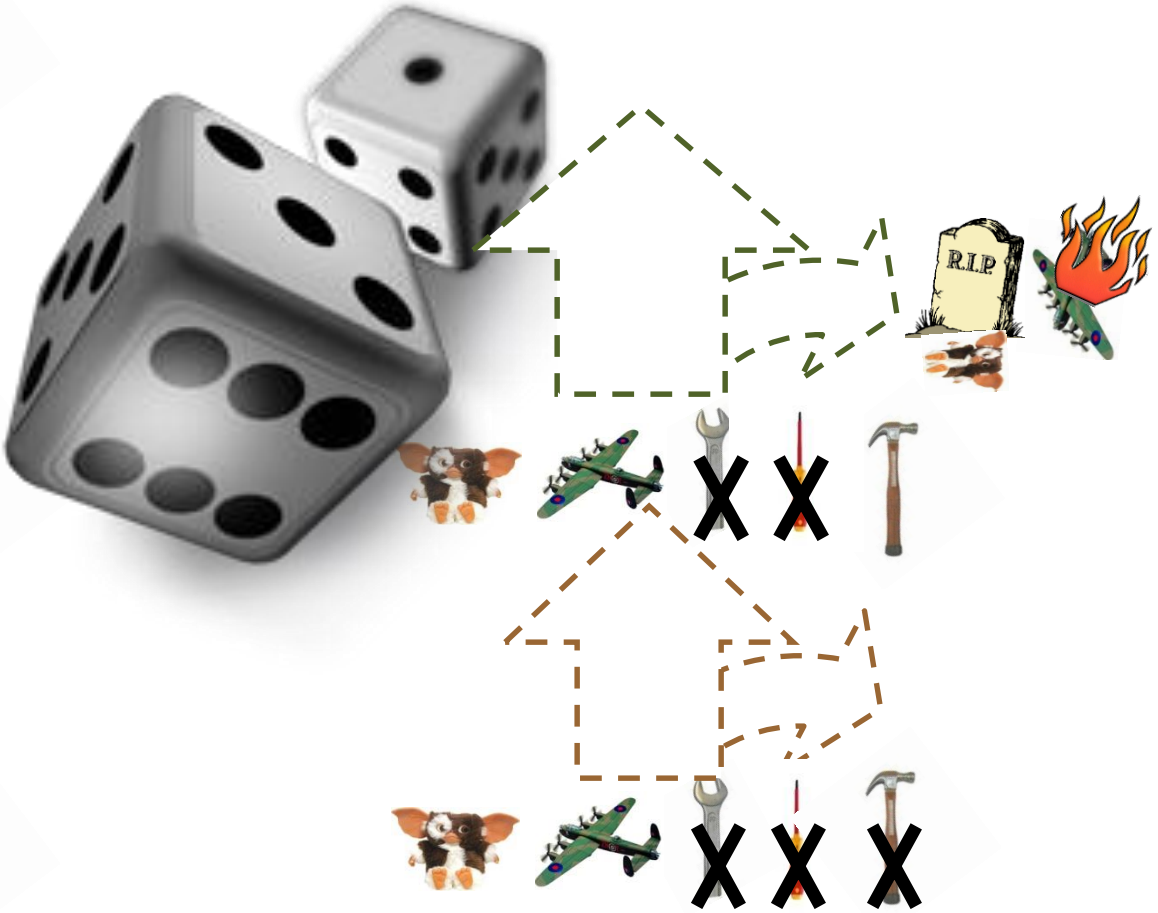
- **In particular, breaks down in the presence of dead ends**
  - Originally designed for MDPs without them

FF-Replan and Dead Ends

Deterministic plan:



Its possible execution:



Putting "Probabilistic" Back Into Planning

- FF-Replan is oblivious to probabilities
  - Its main undoing
  - How do we take them into account?

- **Sample determinizations probabilistically!**
  - Hopefully, probabilistically unlikely plans will be rarely found

- Basic idea behind *FF-Hindsight*

FF-Hindsight: Overview

(Estimating Q-Value, Q(s,a))

S: Current State, A(S) → S'

1. For Each Action A, Draw Future Samples

Each Sample is a Deterministic Planning Problem

2. Solve Time-Dependent Classical Problems

See if you have goal-reaching solutions, estimate Q(s,A)

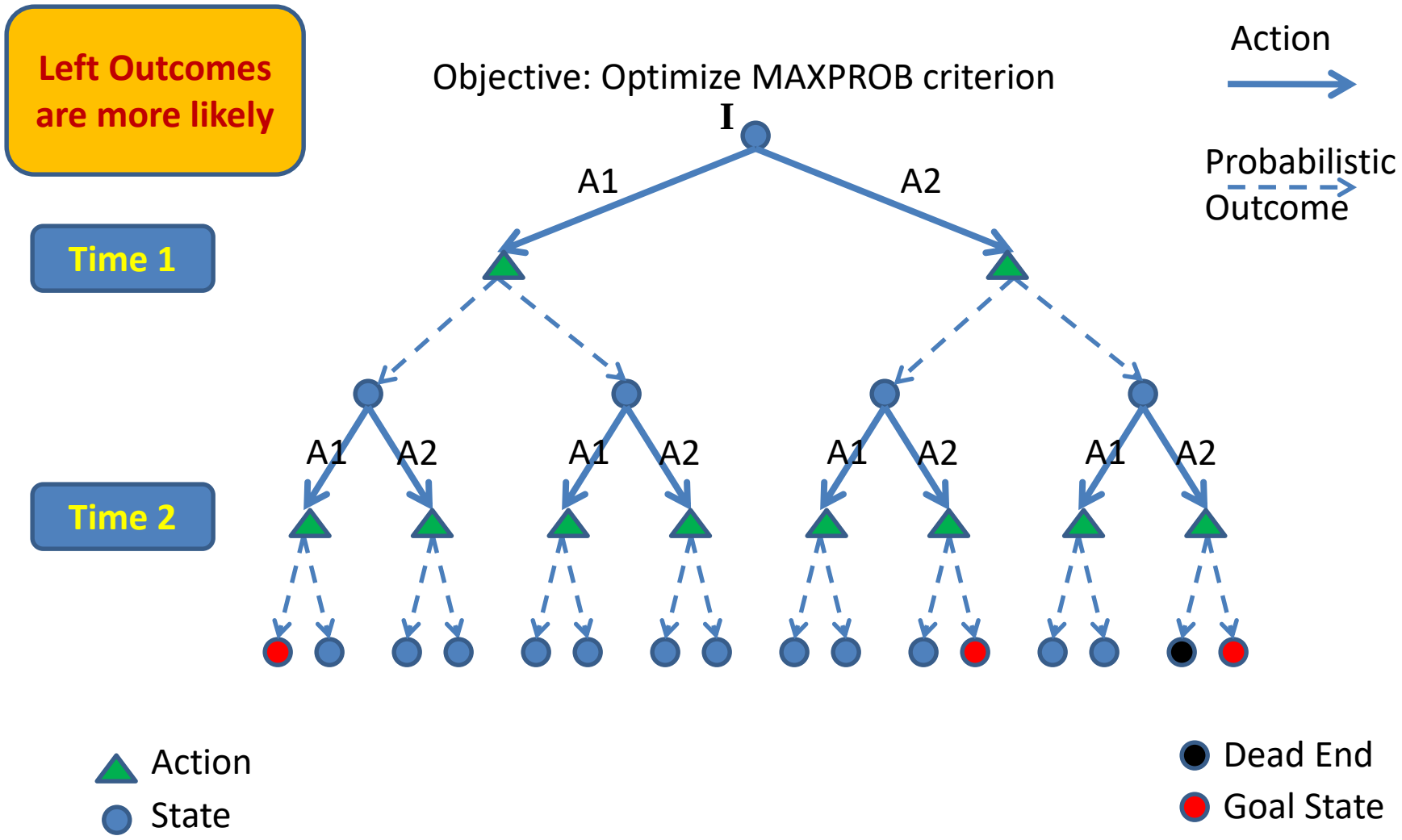
3. Aggregate the solutions for each action

$\text{Max}_A\ Q(s,A)$

4. Select the action with best aggregation

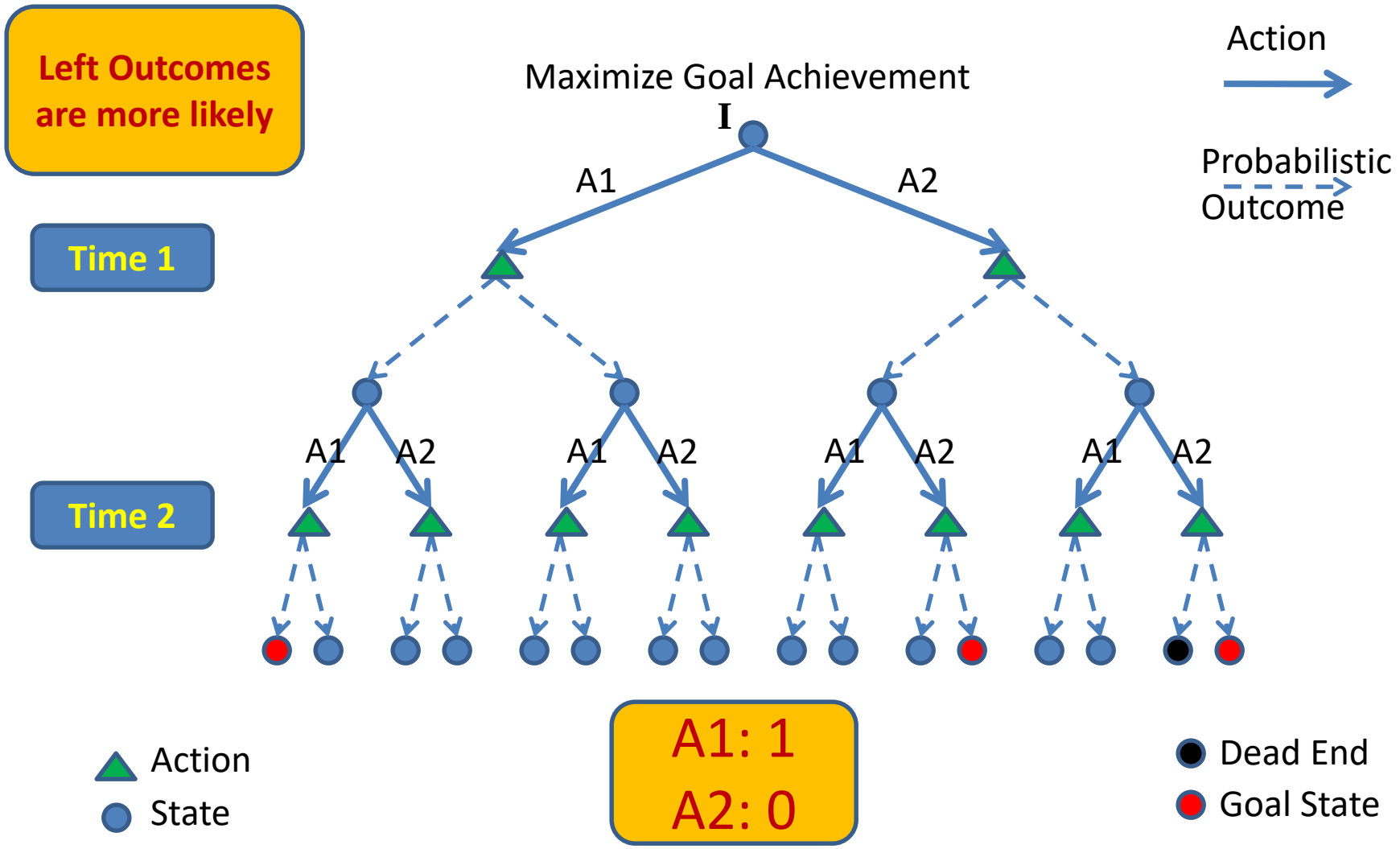
Slide courtesy of S. Yoon, A. Fern, R. Givan, and R. Kambhampati

FF-Hindsight: Example



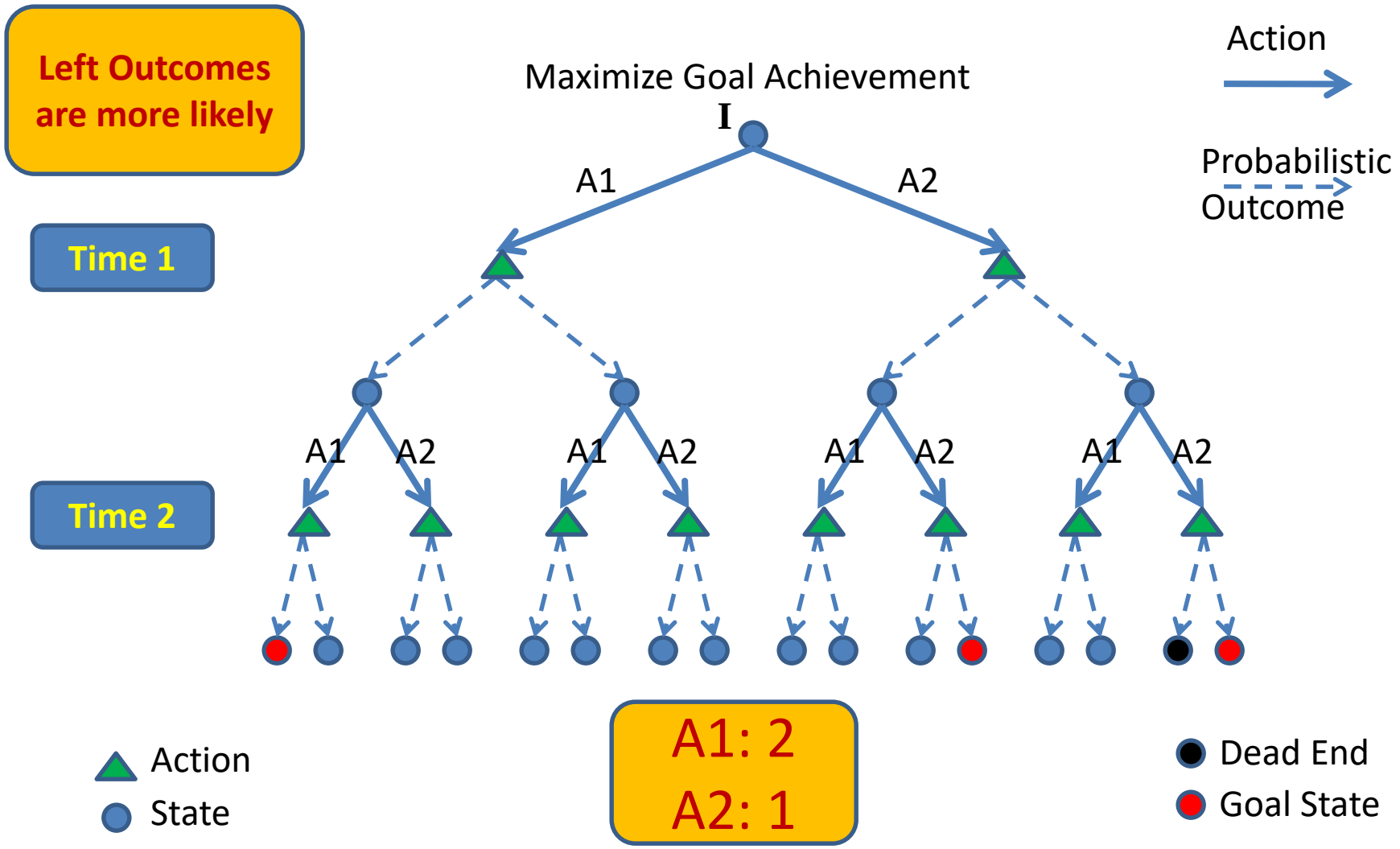
Slide courtesy of S. Yoon, A. Fern, R. Givan, and R. Kambhampati

FF-Hindsight: Sampling a Future-1



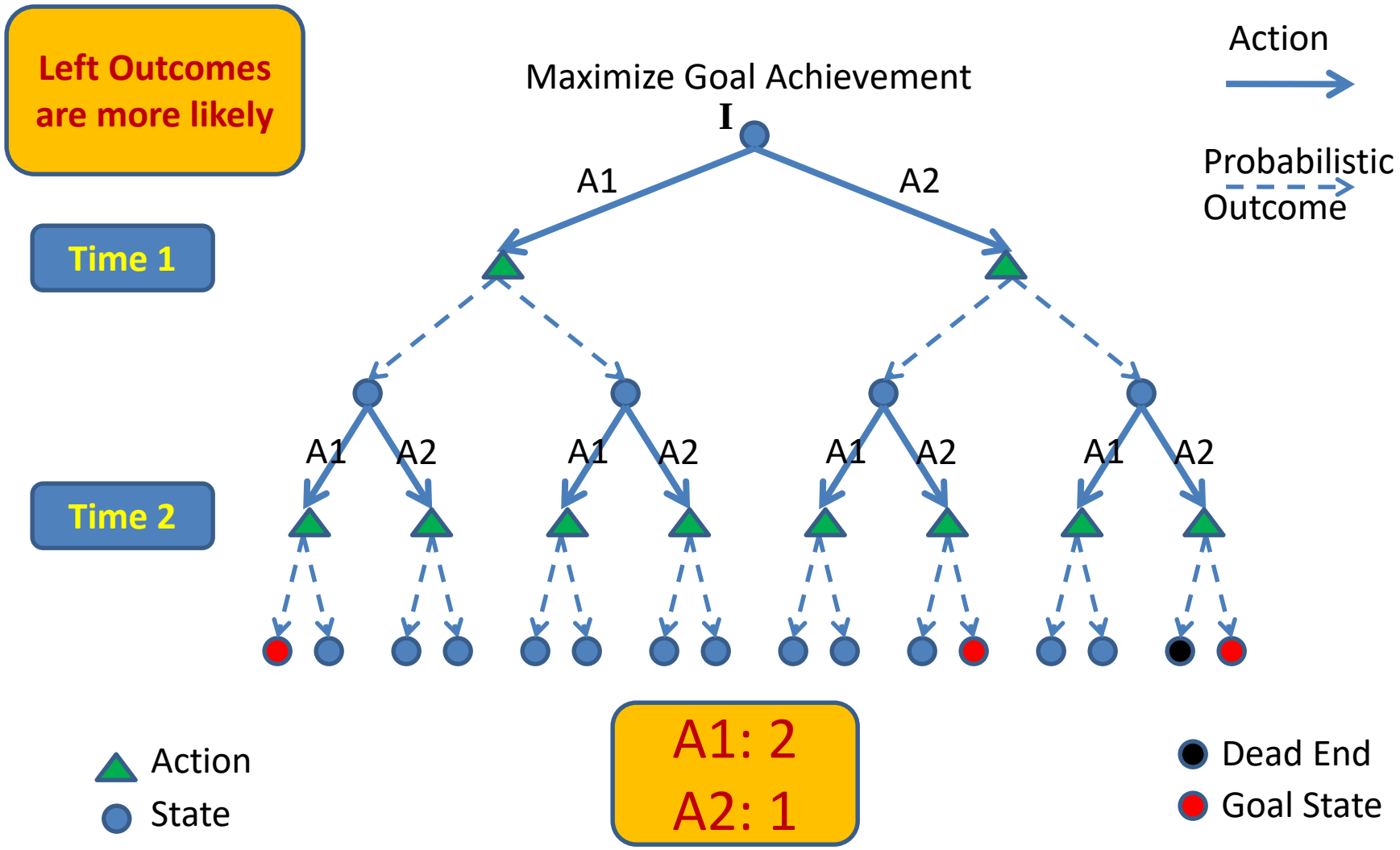
Slide courtesy of S. Yoon, A. Fern, R. Givan, and R. Kambhampati

FF-Hindsight: Sampling a Future-2



Slide courtesy of S. Yoon, A. Fern, R. Givan, and R. Kambhampati

FF-Hindsight: Sampling a Future-3



FF-Hindsight: Details & Theoretical Properties

- For each s, FF-Hindsight samples *w* *L-horizon futures* F^L
  - In factored MDPs, amounts to choosing *a*'s outcome for each *h*
- Futures are solved by the FF planner
  - Fast, since they are much smaller than the AO determinization
- With enough futures, will find MAXPROB-optimal policy
  - If horizon *H* is large enough and a few other assumptions
- **Much better than FF-Replan on MDPs with dead ends**
  - But also slower – lots of FF invocations!

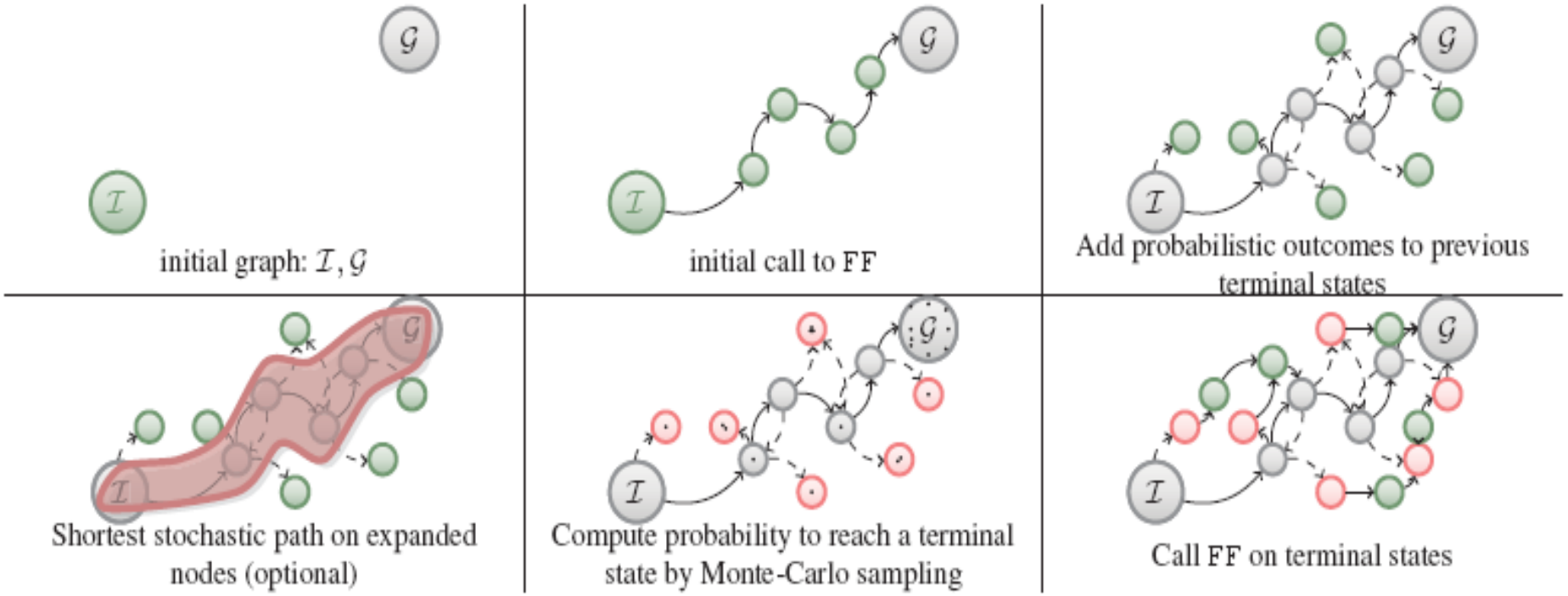
Providing Solution Guarantees

- FF-Replan provides no solution guarantees
  - May have $P_G = 0$ on SSPs with dead ends, even if $P^*_G > 0$
  - Wastes solutions: generates them, then forgets them
- FF-Hindsight provides some theoretical guarantees
  - Practical implementations distinct from theory
  - Wastes solutions: generates them, then forgets them
- **RFF (Robust FF) provides quality guarantees in practice**
  - Constructs a policy tree out of deterministic plans

RFF: Overview

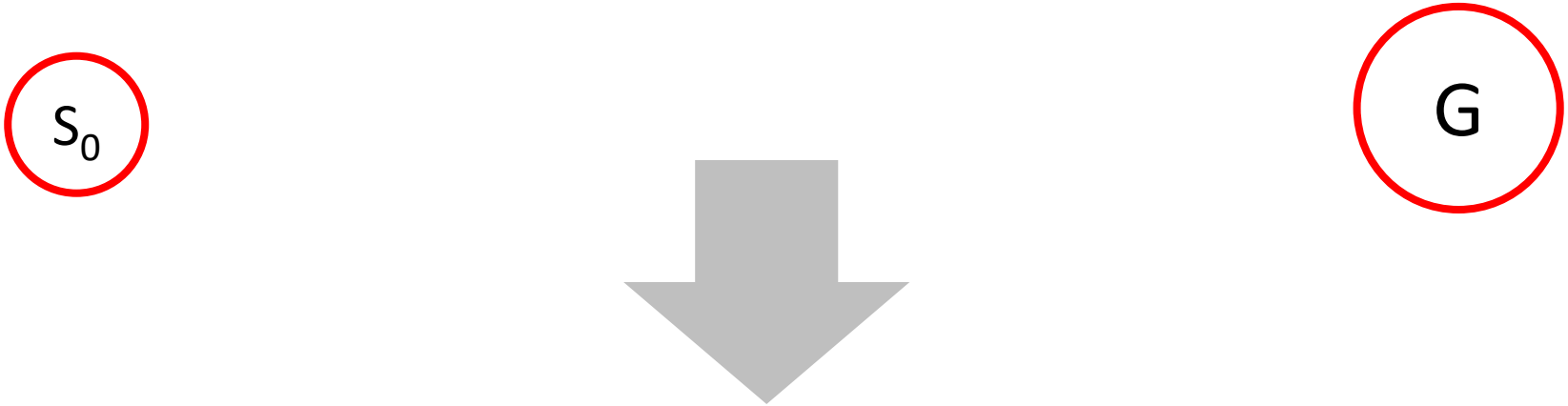
[Teichteil-Königsbuch, Kuter, Infantes, 2010]

Make sure the probability of ending up in an unknown state is < ε



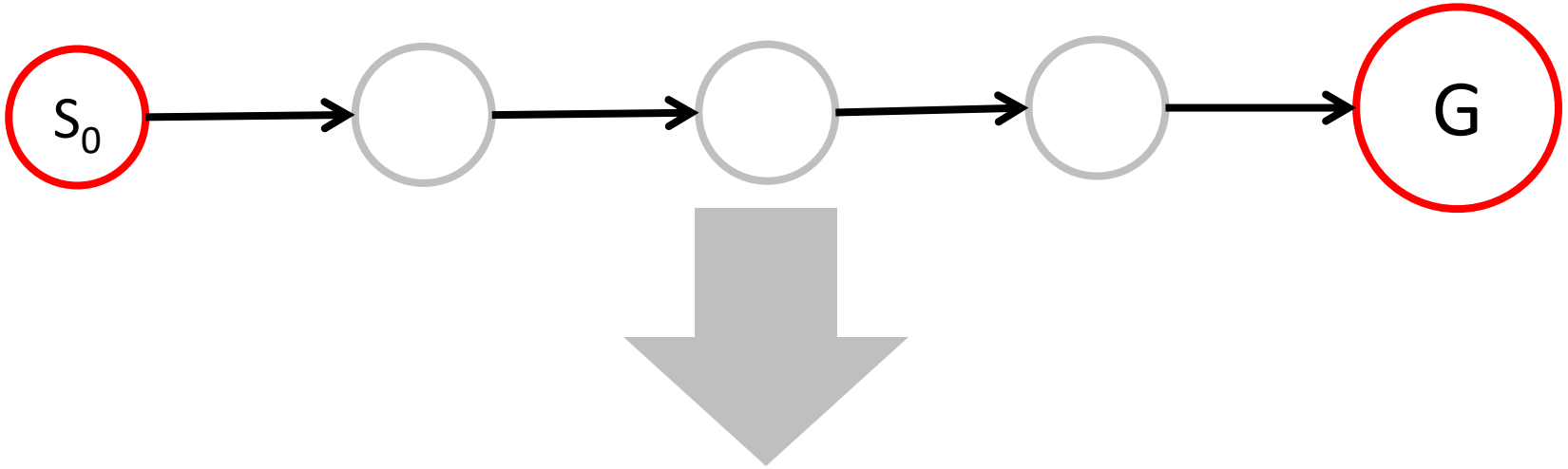
RFF: Initialization

1. Generate either the AO or MLO determinization. Start with the policy graph consisting of the initial state s_0 and all goal states G



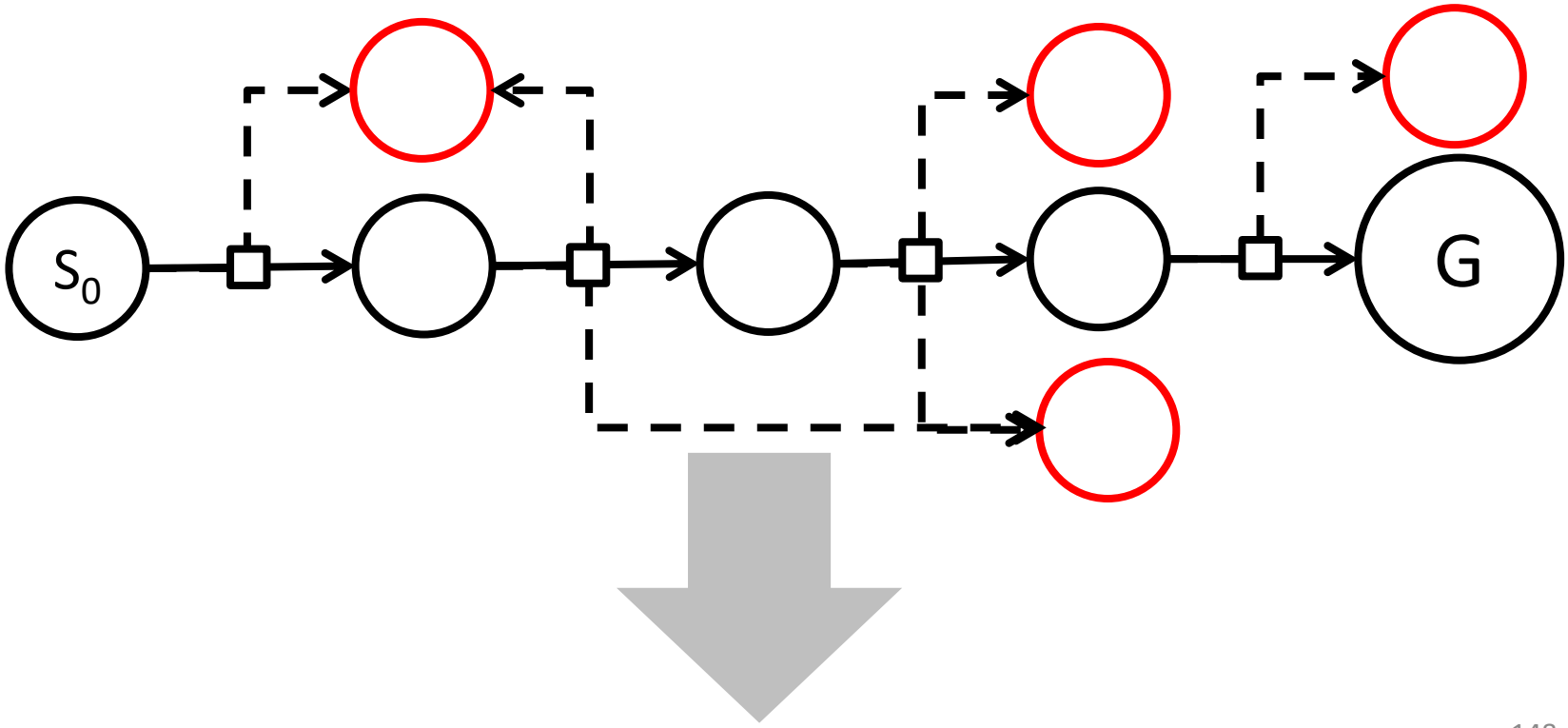
RFF: Finding an Initial Plan

2. Run FF on the chosen determinization and add all the states along the found plan to the policy graph.



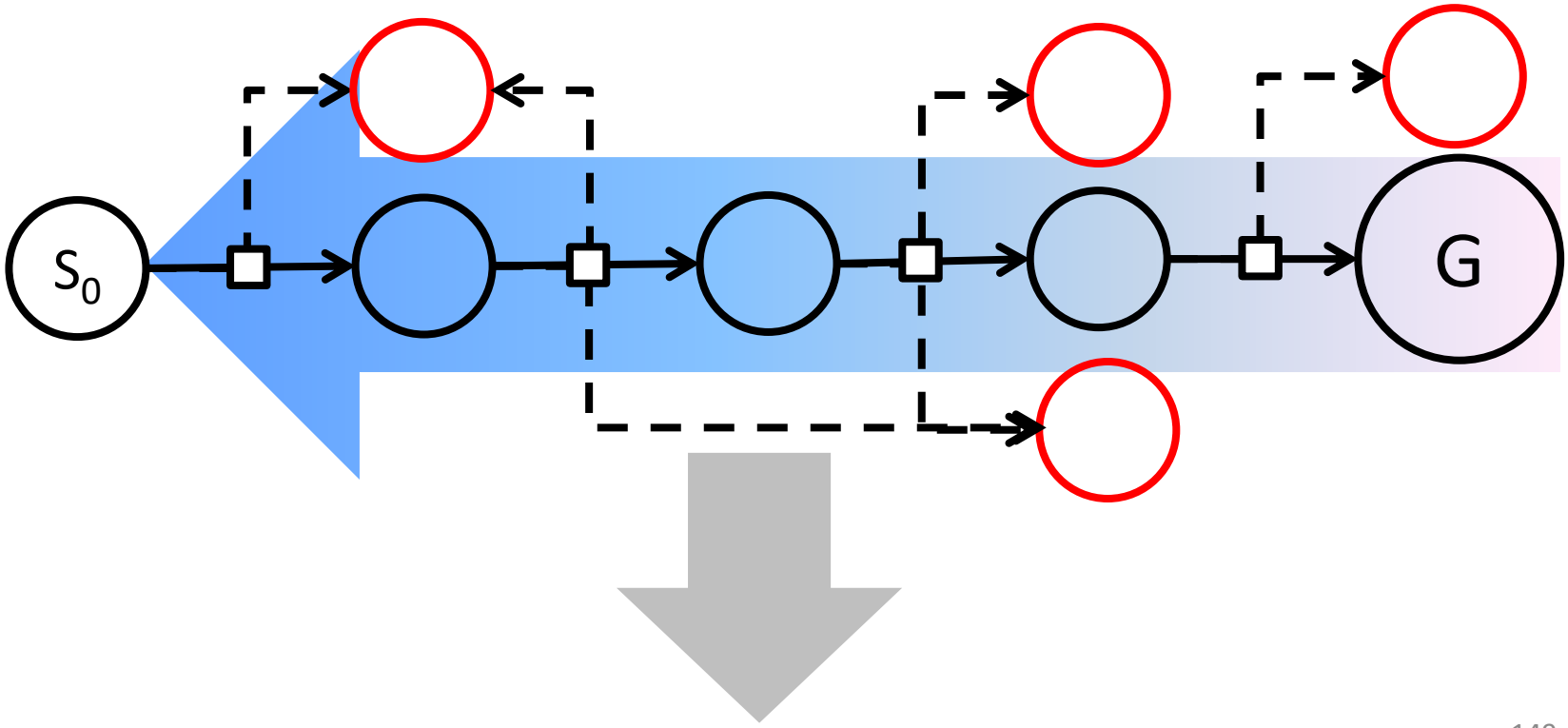
RFF: Adding Alternative Outcomes

3. Augment the graph with states to which other outcomes of the actions in the found plan could lead and that are not in the graph already. They are the policy graph's *fringe states*.



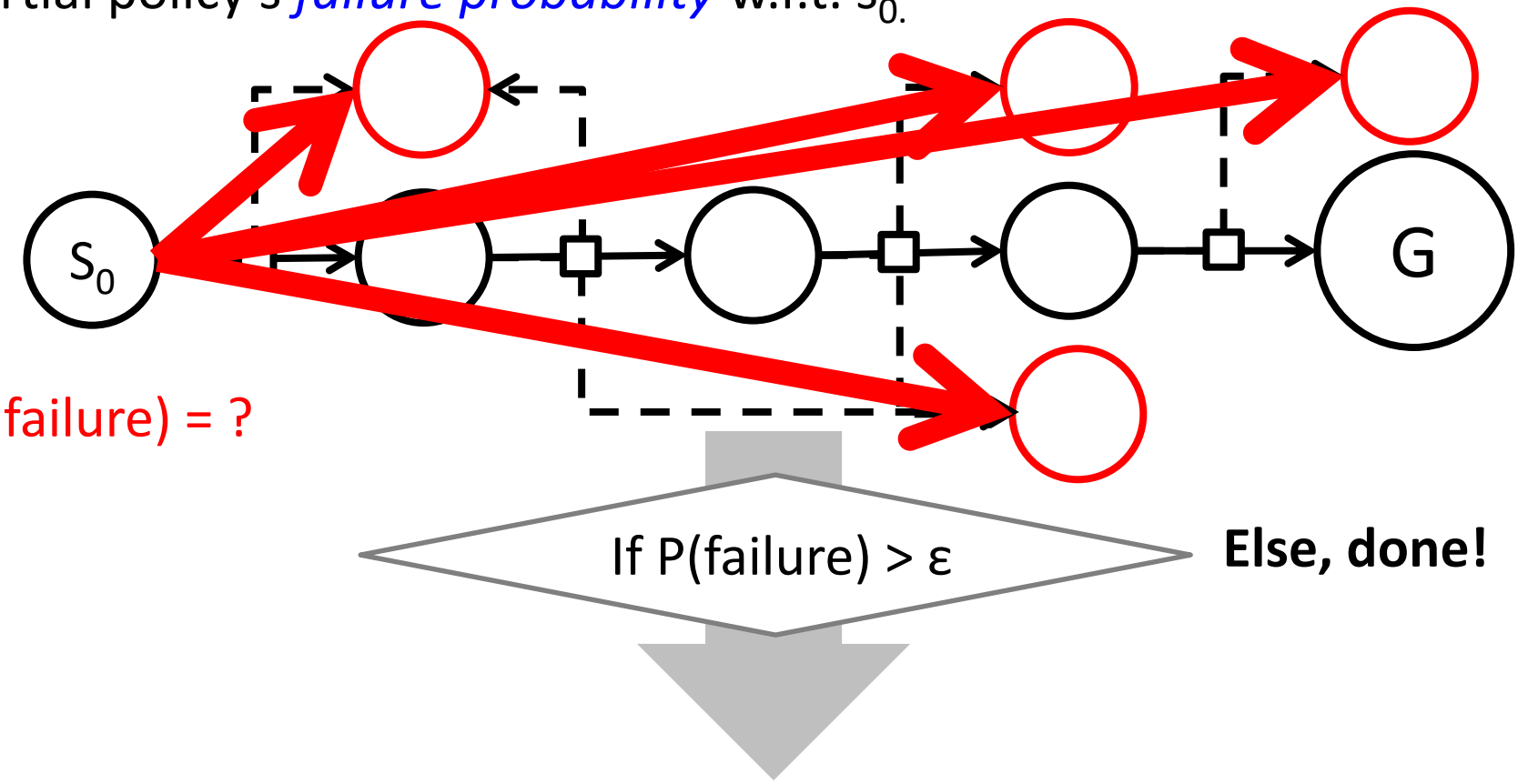
RFF: Run VI **(Optional)**

4. Run VI to propagate heuristic values of the newly added states.
This possibly changes the graph's fringe and helps avoid dead ends!



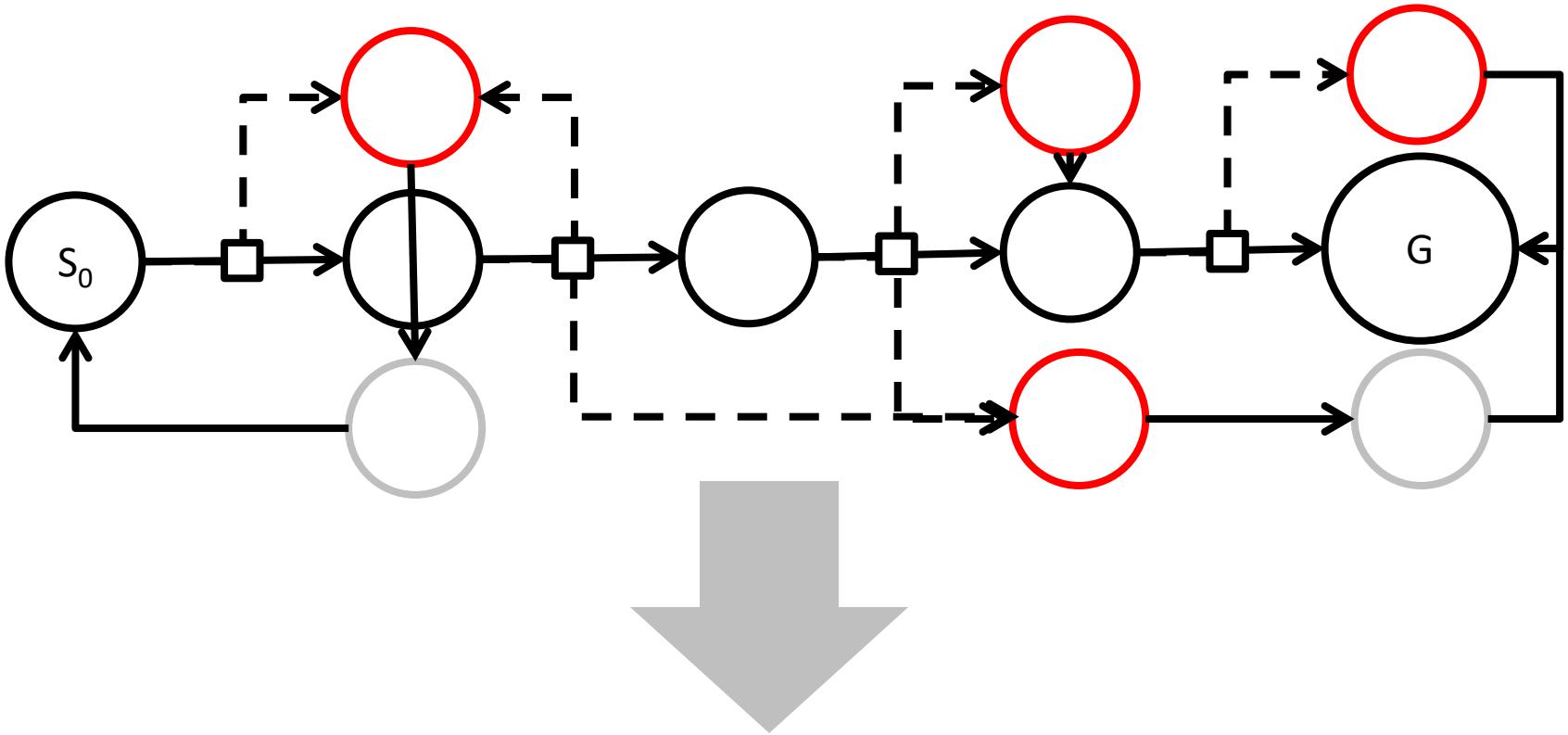
RFF: Computing Replanning Probability

5. Estimate the probability *P(failure)* of reaching the fringe states (e.g., using Monte-Carlo sampling) from s_0. This is the current partial policy's *failure probability* w.r.t. s_0.



RFF: Finding Plans from the Fringe

6. From each of the fringe states, run FF to find a plan to reach the goal or one of the states already in the policy graph.



Go back to step 3: Adding Alternative Outcomes

RFF: Theoretical Properties

- Fast
  - FF-Replan forgets computed policies
  - RFF essentially memorizes them

- **When using AO determinization, guaranteed to find a policy that with $P = 1 - \varepsilon$ will not need replanning**

Summary of Determinization Approaches

- Revolutionized SSP MDPs approximation techniques
  - Harnessed the speed of classical planners
  - Eventually, "learned" to take into account probabilities
  - Help optimize for a "proxy" criterion, MAXPROB
- Classical planners help by quickly finding paths to a goal
  - Takes "probabilistic" MDP solvers a while to find them on their own
- **However…**
  - Still almost completely disregard *the expected cost* of a solution
  - Often assume uniform action costs (since many classical planners do)
  - So far, not useful on *FH* and *IHDR* MDPs turned into *SSPs*
    - Reaching a goal in them is trivial, need to approximate reward more directly
  - Impractical on problems with large numbers of outcomes

Outline

- Online Algorithms
  - FF Replan
  - FF Hindsight
  - RFF
- Offline Algorithms
  - Inadmissible Heuristics
  - Dimensionality Reduction
  - Other Determinization Approaches

Moving on to Approximate Offline Planning

- Useful when there is no time to plan as you go …
  - E.g., when playing a fast-paced game

Inadmissible Heuristic Search

- Why?
  - May require less space than admissible heuristic search

The FF Heuristic

[Hoffmann and Nebel, 2001]

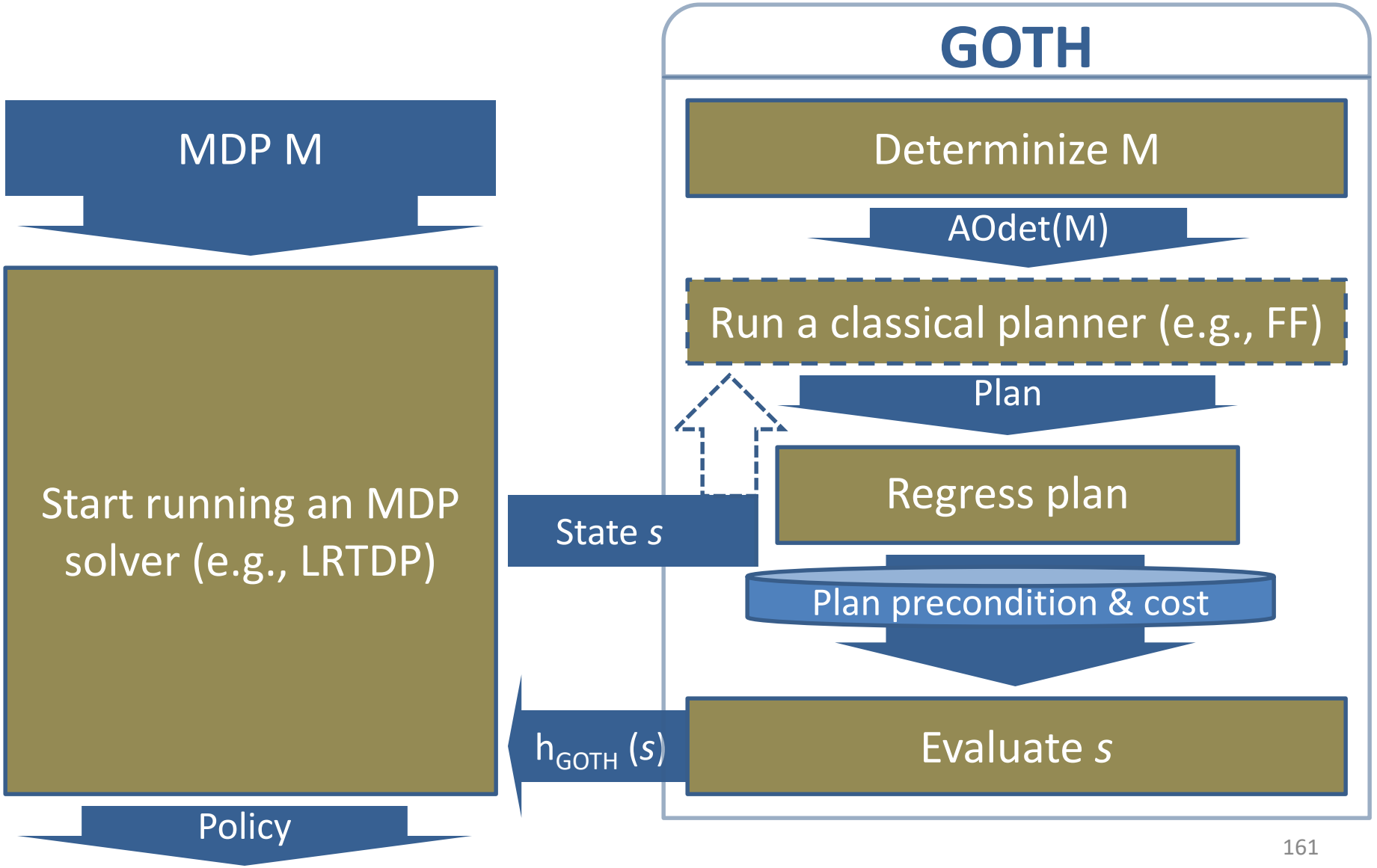
- Taken directly from deterministic planning
  - A major component of the formidable FF planner
- Uses the all-outcome determinization of a PPDDL MDP
  - But ignores the *delete effects* (negative literals in action outcomes)
  - Actions never "unachieve" literals, always make progress to goal
- **$h_{FF}(s)$ = approximate cost of a plan from *s* to a goal in the delete relaxation**
- Very fast due to using the delete relaxation
- Very informative

The GOTH Heuristic

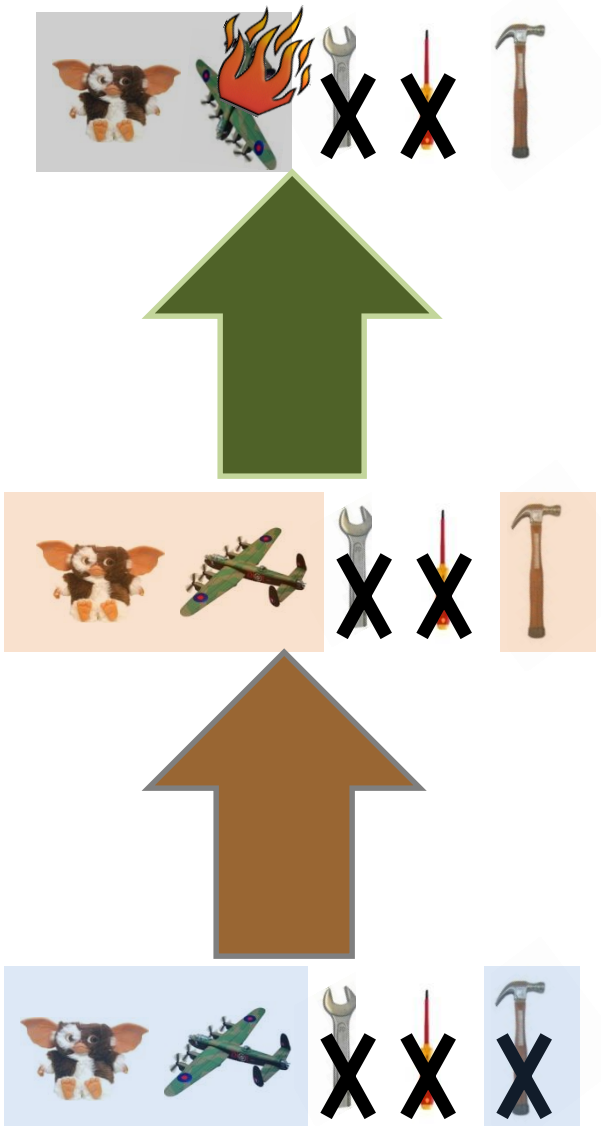
[Kolobov, Mausam, Weld, 2010]

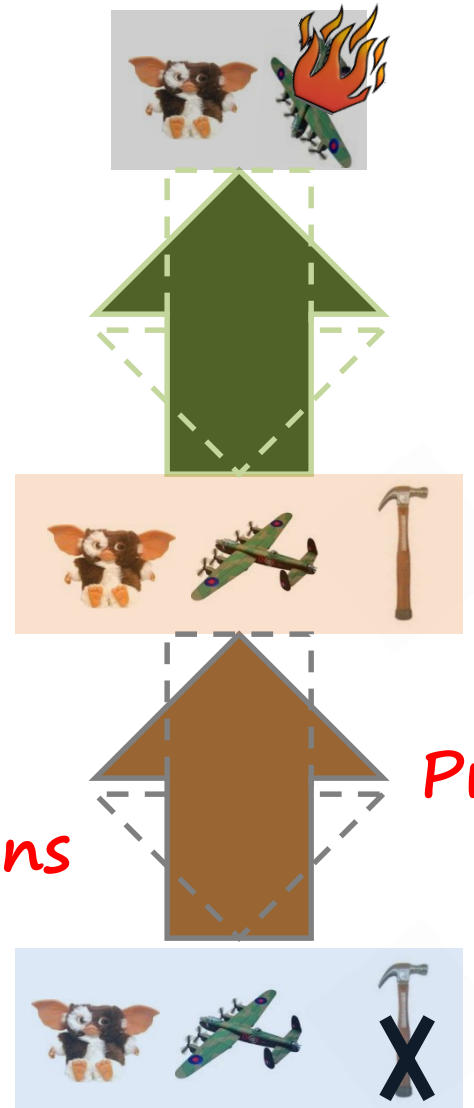
- Designed for MDPs at the start (not adapted classical)
- Motivation: would be good to estimate *h(s)* as cost of a non-relaxed deterministic goal plan from *s*
  - But too expensive to call a classical planner from every *s*
  - Instead, call from only a few *s* and generalize estimates to others
- Uses AO determinization and the FF planner

GOTH Overview



Regressing Trajectories





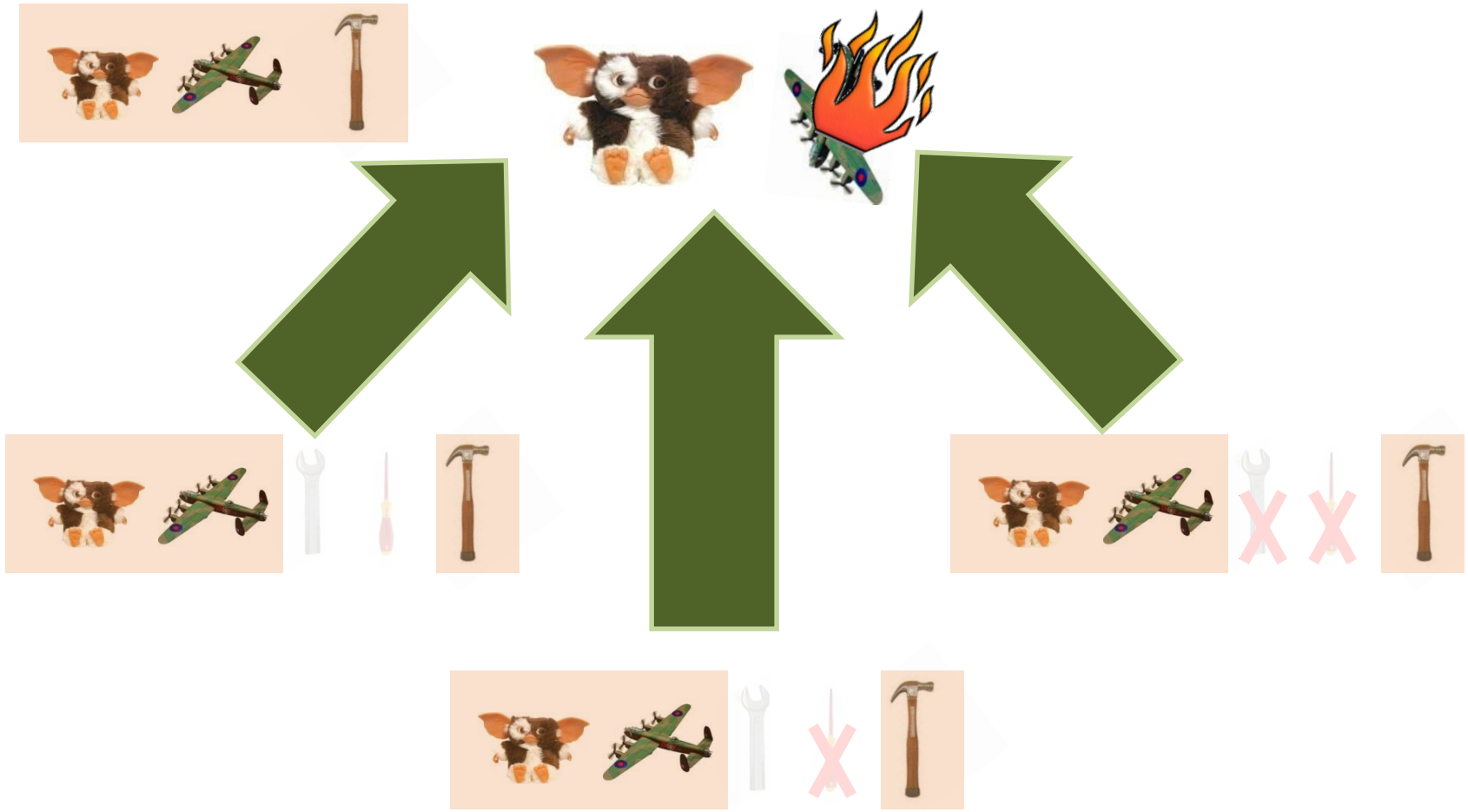
= 1

= 2

Plan preconditions

Precondition costs

Plan Preconditions



Estimating State Values

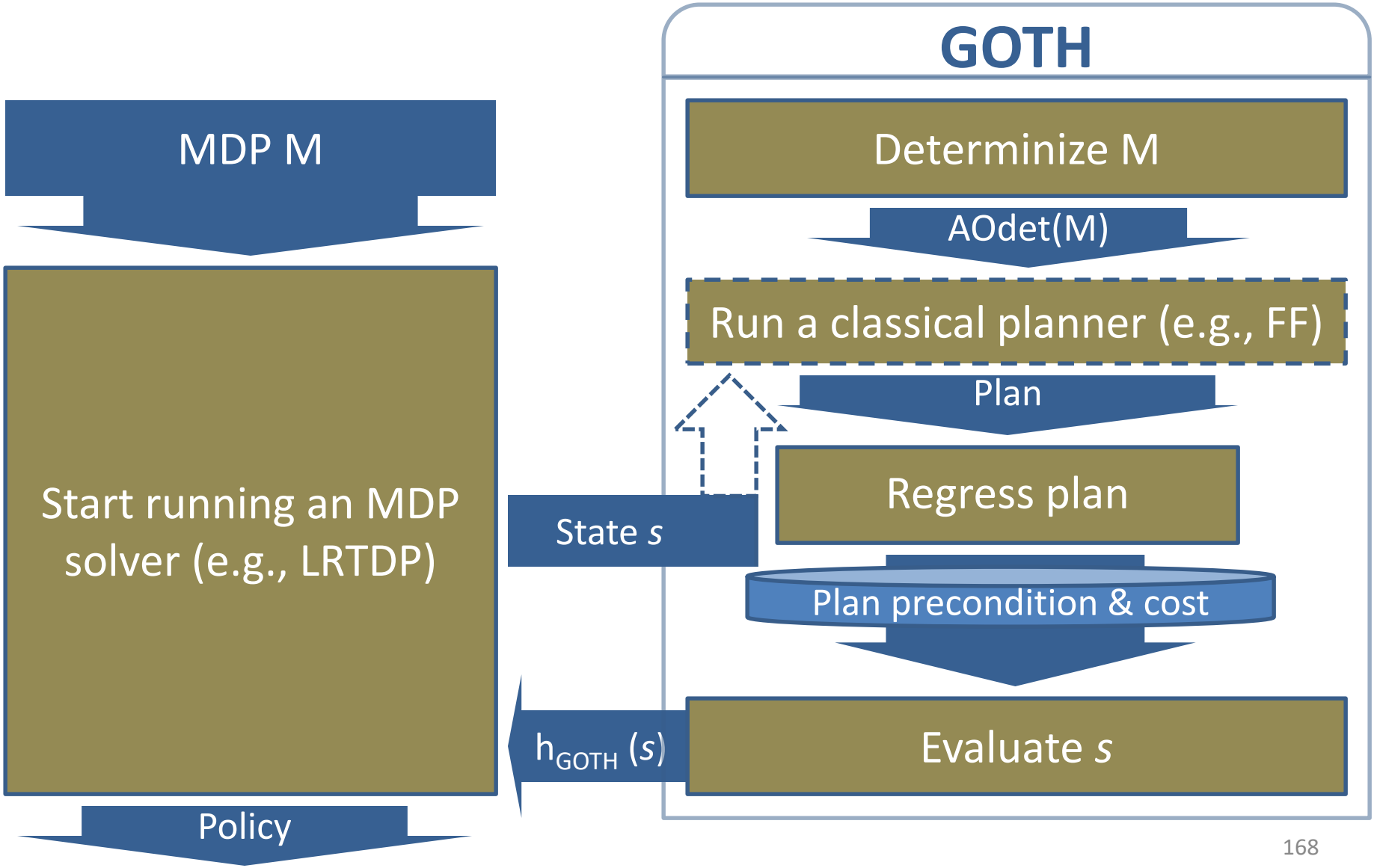
- Intuition
  - Each plan precondition cost is a "candidate" heuristic value

- Define $h_{GOTH}(s)$ as MIN of all available plan precondition values applicable in *s*
  - If none applicable in *s*, run a classical planner and find some
  - Amortizes the cost of classical planning across many states

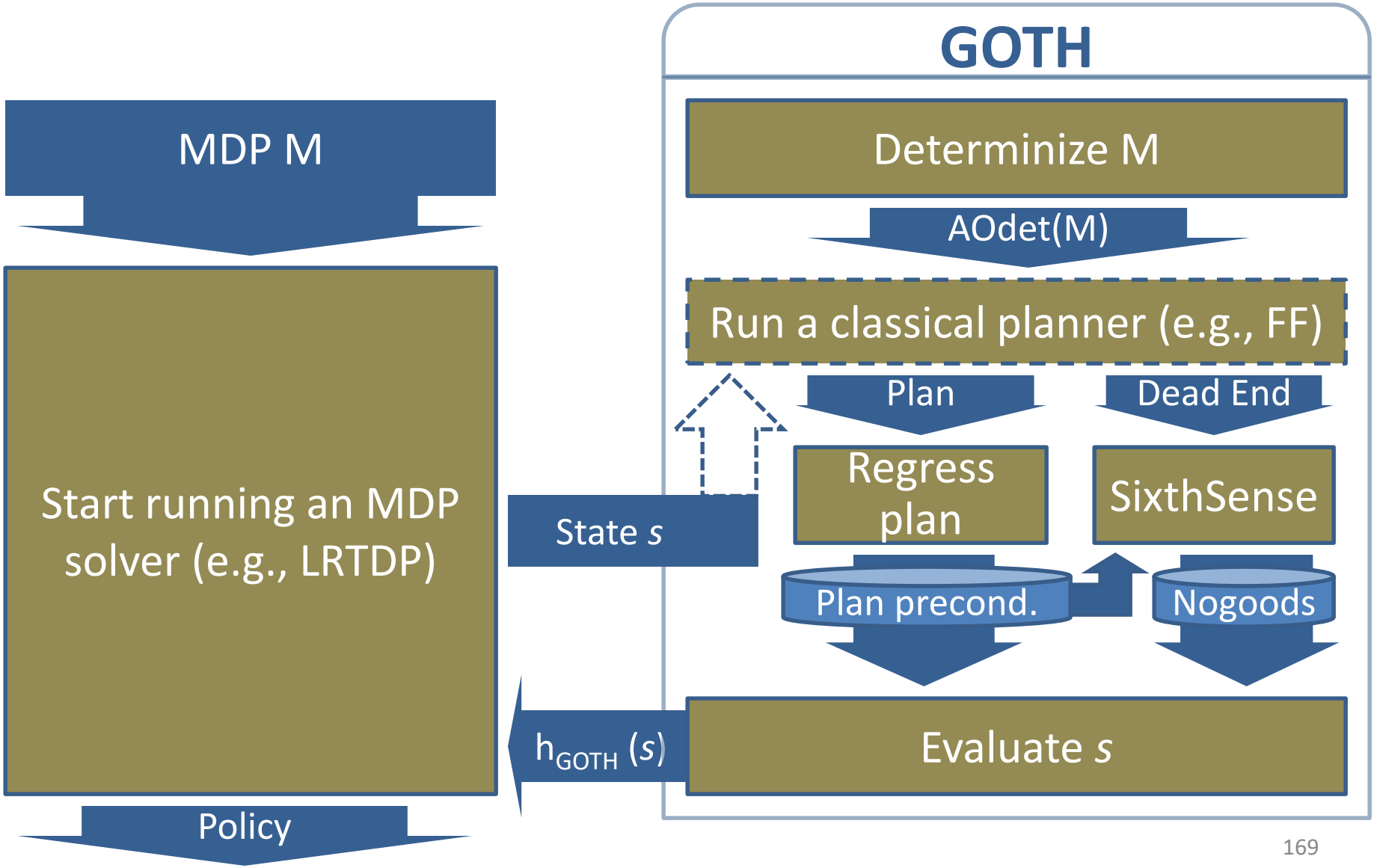
What about Dead Ends?

- How to find an implicit dead-end state?
  - FF doesn't return a solution (say in some fixed time)
- GOTH generalizes each successful trajectory.
  - Can we generalize each implicit dead-end state?
- SixthSense!

GOTH Overview



GOTH+6S Overview

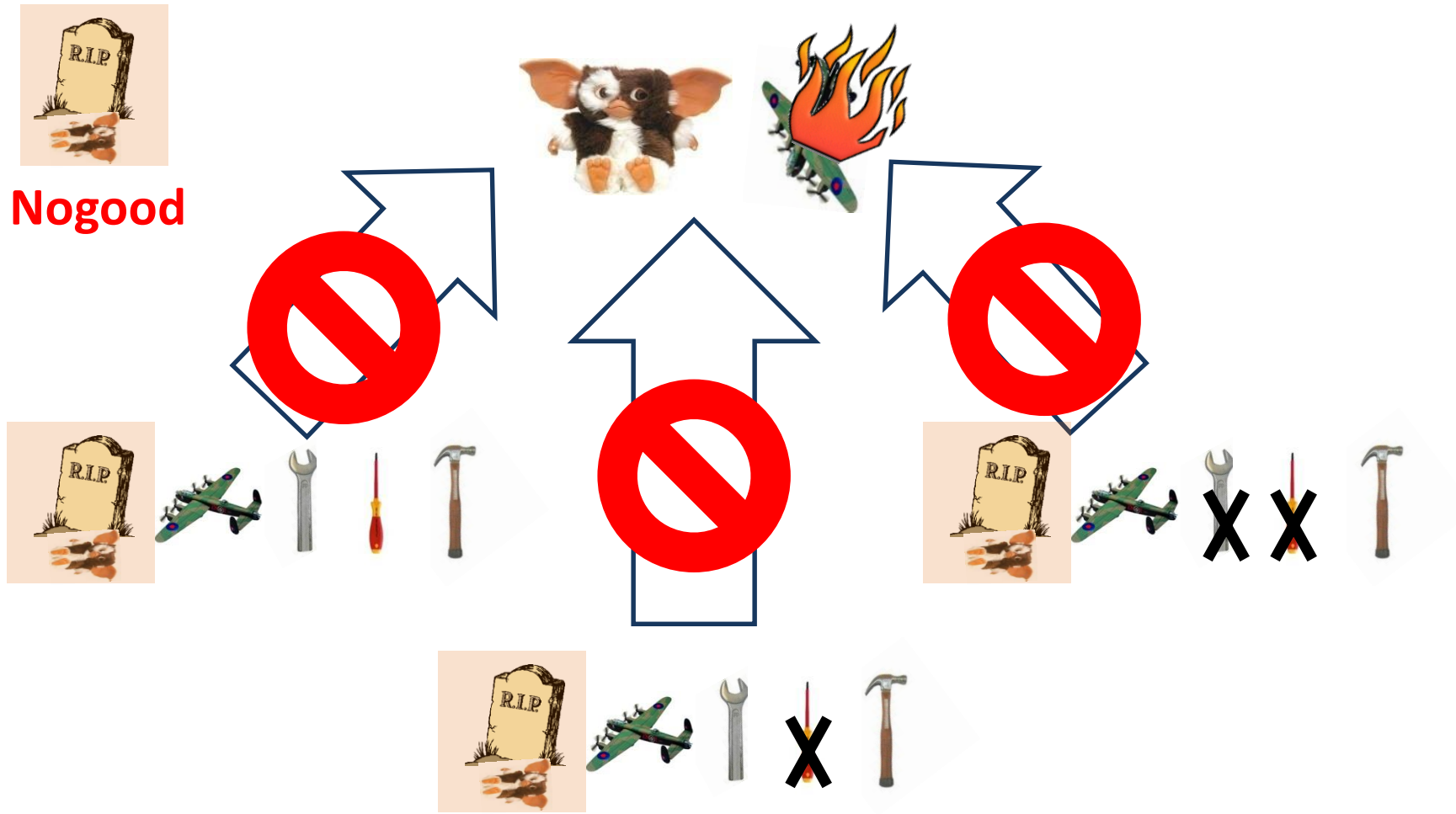


Research Question

Can we devis  **dentification procedure fas memoization?**

Learns feature combinations whose presence guarantees a state to be a dead end

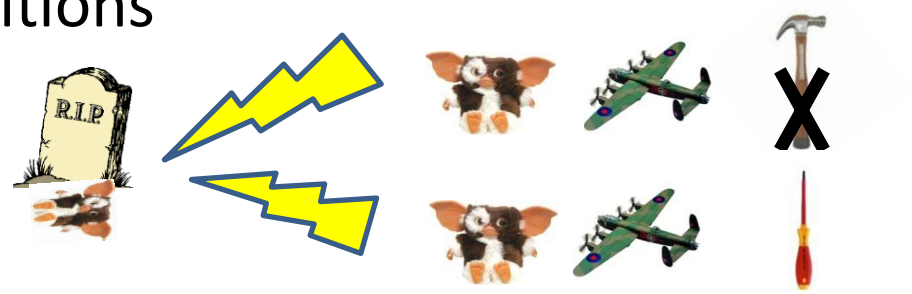
Nogoods



Generate-and-Test Procedure

[Kolobov, Mausam, Weld 2010]

- **Generate a nogood candidate**
  - Key insight: Nogood = conjunction that *defeats* all known plan preconditions



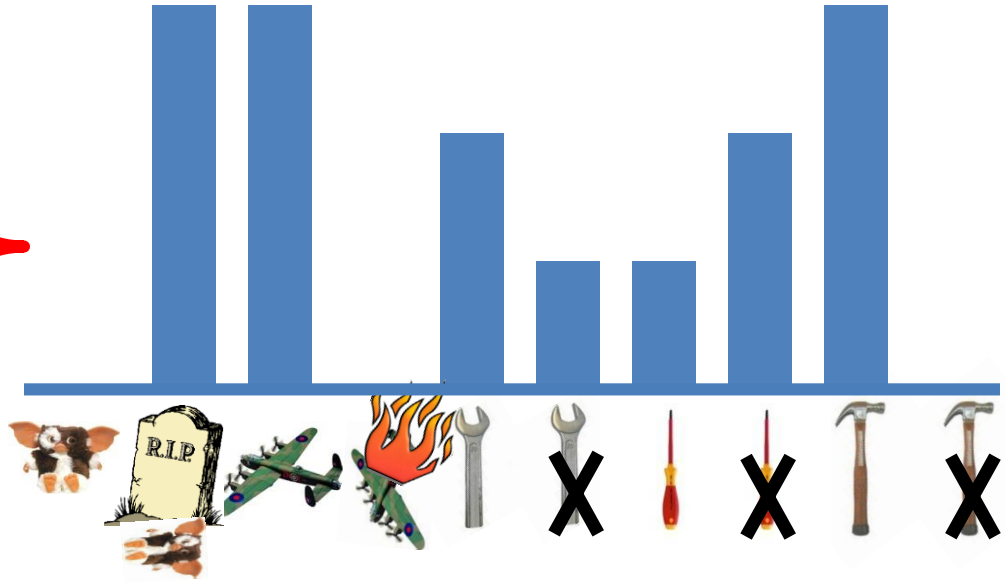
  - For each plan precndition, pick a literal that defeats it

- **Test the candidate**
  - Needed for soundness, since we don't know all preconditions
  - Use the non-relaxed Planning Graph algorithm

Generating a Nogood Candidate

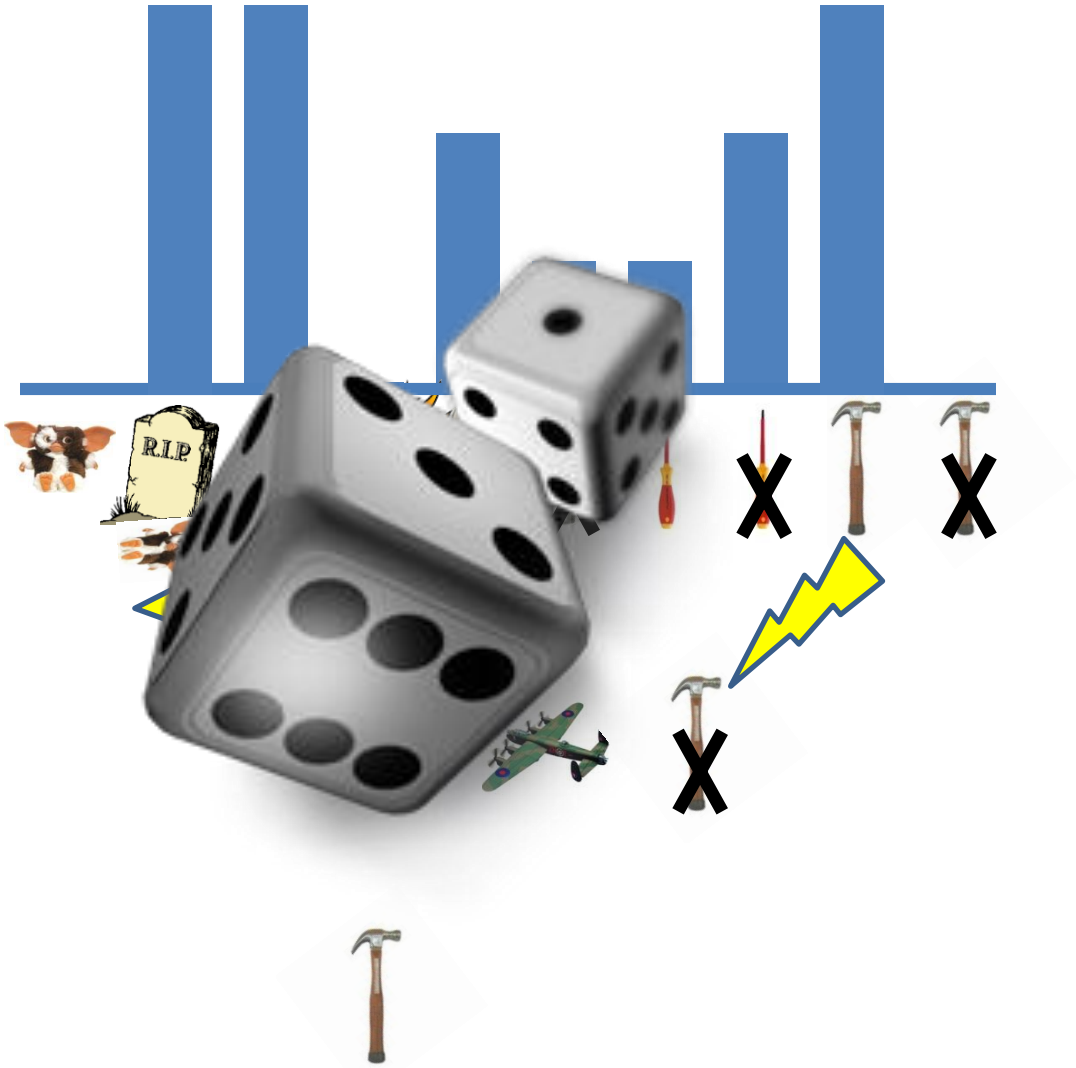
Compute histogram of literal occurrence in dead ends:

Training dead ends:



Generating a Nogood Candidate

Dead-end literal occurrence stats

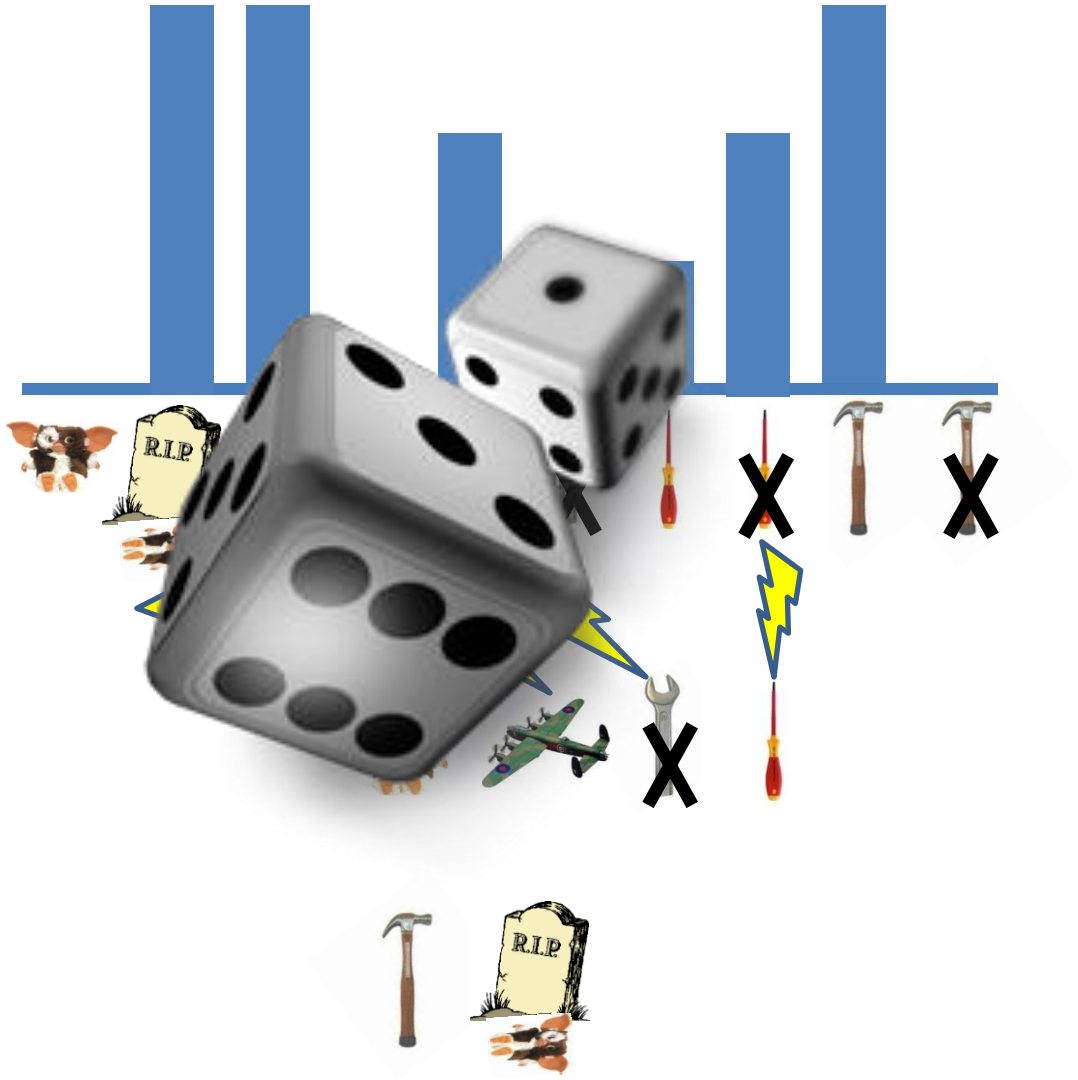


Current basis function

Current nogood candidate

Generating a Nogood Candidate

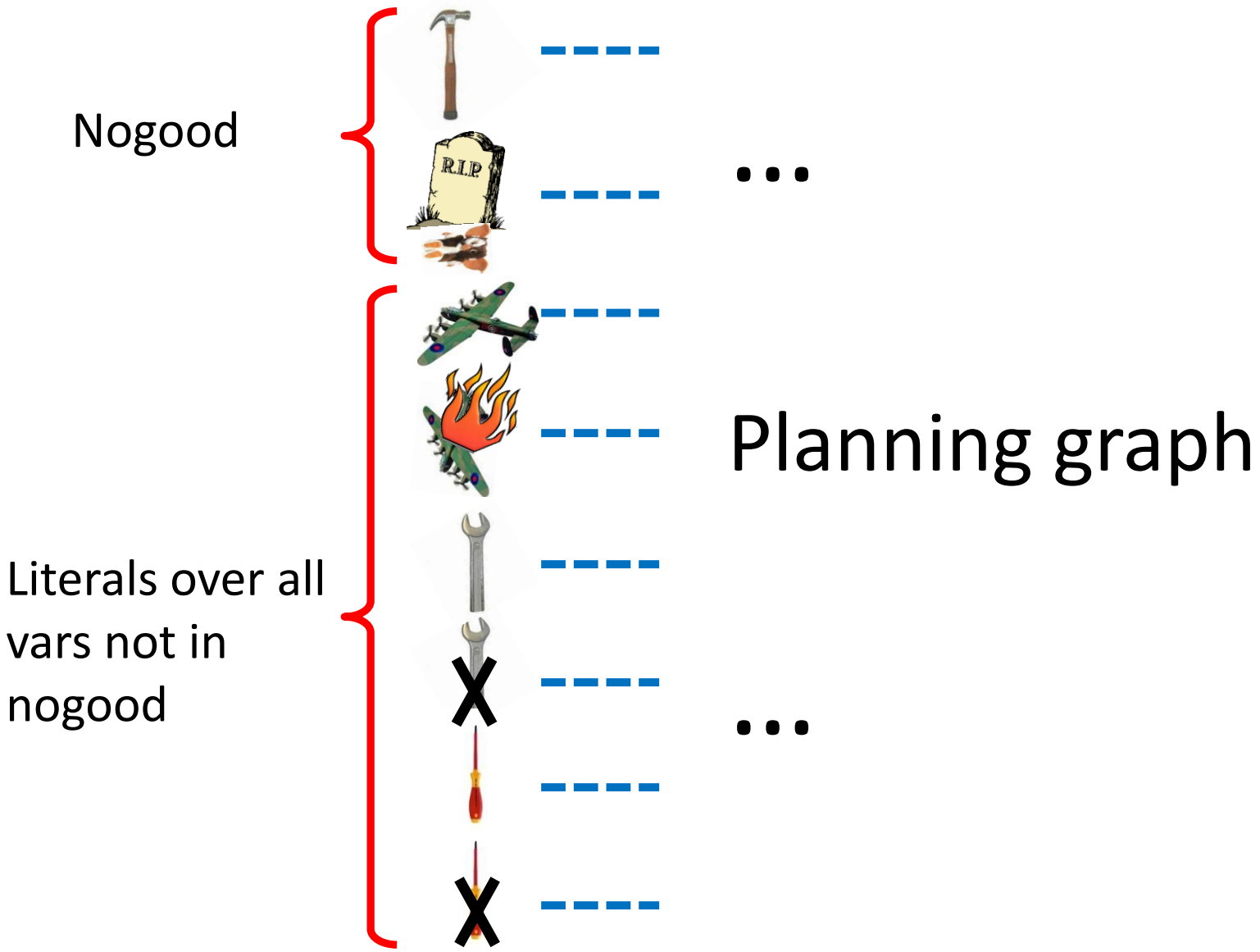
Dead-end literal occurrence stats



Current basis function

Current nogood candidate

Testing the Candidate



Testing the Candidate

- If the Planning Graph fails to reach the goal, the candidate is a nogood.
  - Planning Graph is complete, hence this is sound

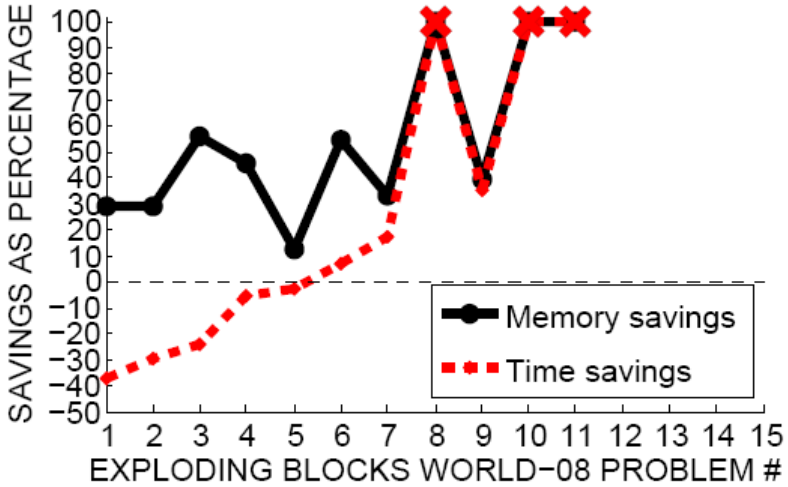
- Note:  in   is superfluous
  - Remove literals one-by-one and test as above to get

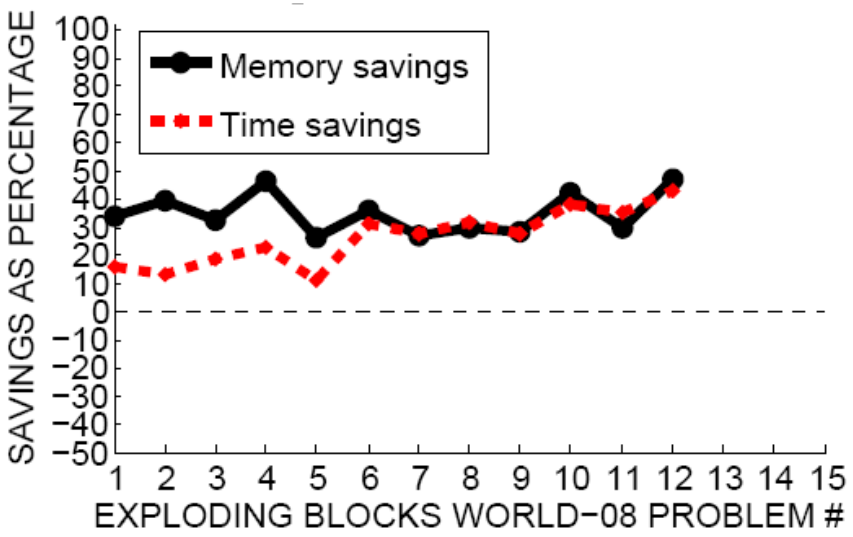
Scheduling

- Need to invoke learning more than once
- Never know how much training data is “enough”
- Solution: adaptive scheduler
  - Finds a “good” amount of training data, invokes learning accordingly

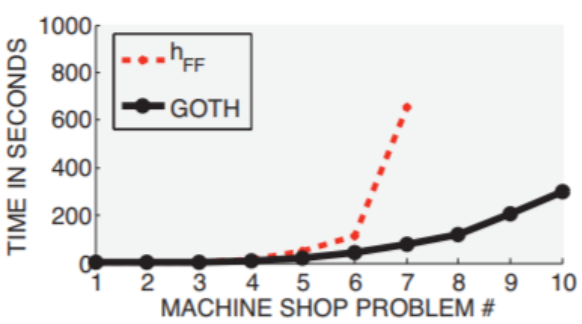
Benefits of SixthSense

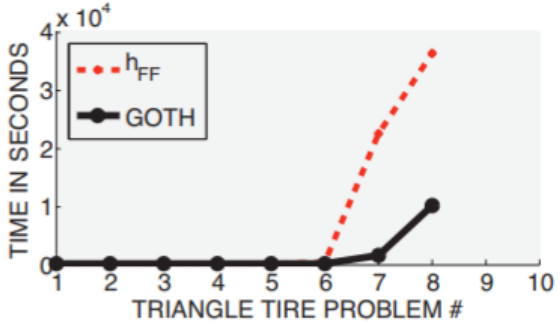
- **Can act as submodule of many planners and ID dead ends**
  - By checking discovered nogoods against every state





h_{FF} vs GOTH+6S





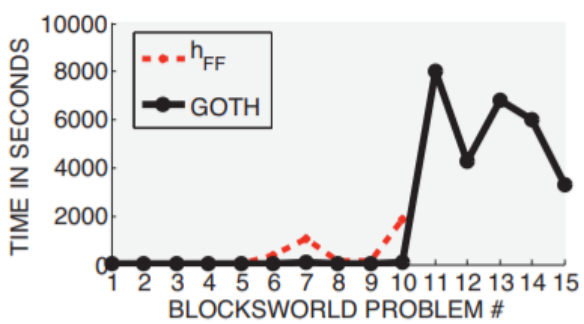
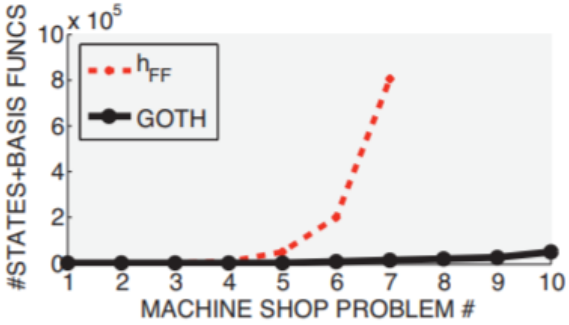
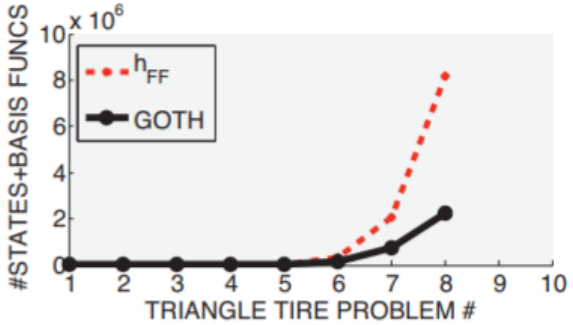


Figure 1: GOTH outperforms h_{FF} on Machine Shop, Triangle Tireworld, and Blocksworld by a large margin both in speed...





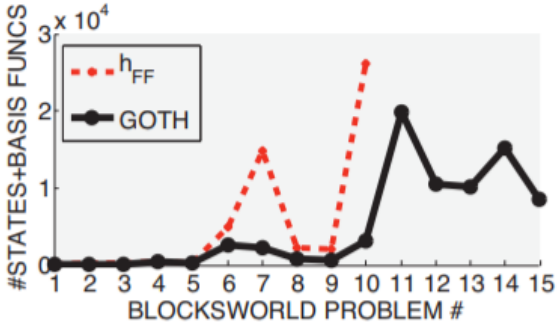


Figure 2: ... and in memory

Outline

- Online Algorithms
  - FF Replan
  - FF Hindsight
  - RFF
- Offline Algorithms
  - Inadmissible Heuristics
  - Dimensionality Reduction
  - Other Determinization Approaches

Previous Work

ReTrASE

- Determinization
  - Determinize the MDP
  - Classical planners ***fast***
  - E.g., FF-Replan
  - Cons: may be troubled by
    - Complex contingencies
    - Probabilities
- Function Approximation
  - Dimensionality reduction
  - Represent state values with basis functions
    - E.g., $V^*(s) \approx \sum_i w_i b_i(s)$
  - Cons:
    - Need a human to get b_i

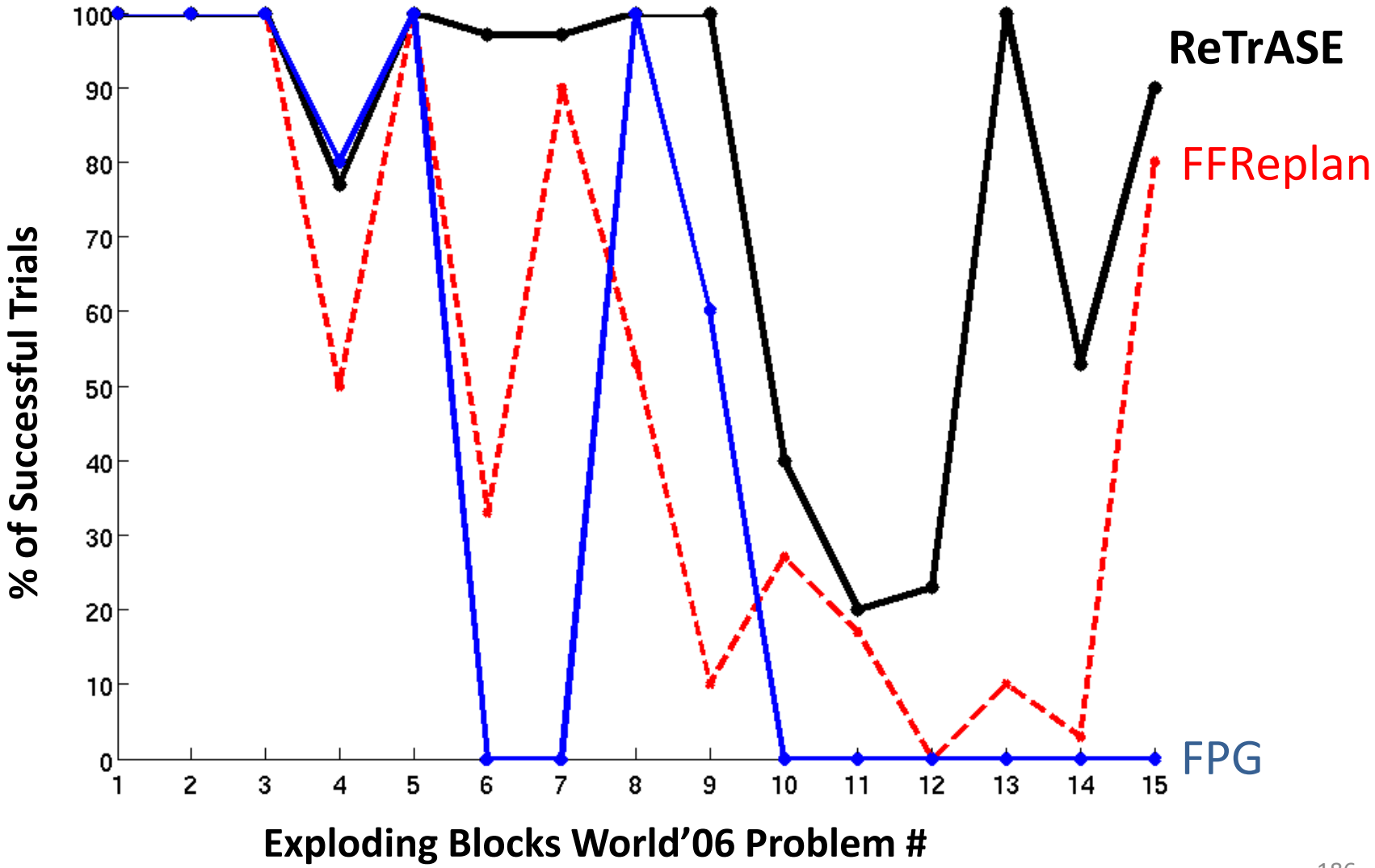
Marry these paradigms to extract problem-specific structure in a fast, problem-independent way.

ReTrASE

[Kolobov, Mausam, Weld, 2009]

- Largely similar to h_{GOTH}
  - Uses preconditions of deterministic plan to evaluate states
- For each plan precondition *p*, defines a *basis function*
  - $B_p(s) = 1$ iff *p* holds in *s*, ∞ otherwise
- Represents $V(s) = min_p\ w_p B_p(s)$
  - Thus, the parameters are w_p for each basis function
  - Problem boils down to learning w_p
  - Does this with modified *RTDP*
- **Crucial observation: # plan preconditions sufficient for representing *V* is typically much smaller than |*S*|**
  - Because one plan precondition can hold in several states
  - Hence, the problem dimension is reduced!

Exploding Blocks World: Success Rate



ReTrASE Theoretical Properties

- Empirically, gives a large reduction in memory vs LRTDP
- Produces good policies (in terms of MAXPROB) when/if converges
- Not guaranteed to converge (weights may oscillate)
- No convergence detection/stopping criterion

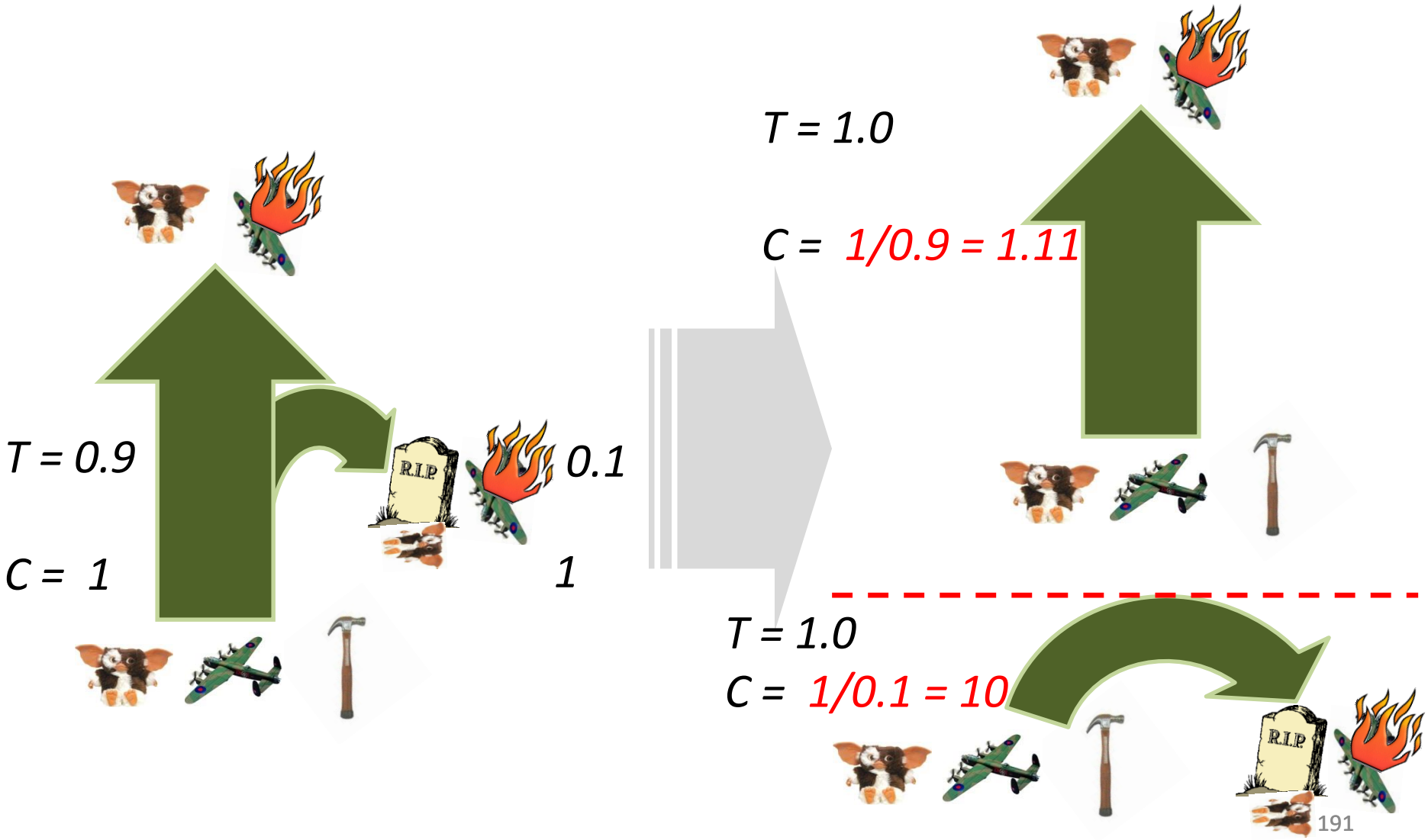
Current Trend: Deep Probabilistic Planning

- Use deep RL ideas
  - for PPDDL or RDDL planning
- Ideas
  - use given model effectively
  - transfer between problem instances

Outline

- Online Algorithms
  - FF Replan
  - FF Hindsight
  - RFF
- Offline Algorithms
  - Inadmissible Heuristics
  - Dimensionality Reduction
  - Other Determinization Approaches

Self-Loop Determinization



Self-Loop Determinization

- Like AO determinization, but modifies action costs
- **“Unlikely” deterministic plans look expensive in SL det.!**
- Used in the HMDPP planner

Space of Determinizations

[Pineda & Zilberstein, 2017]

- Extreme 1
  - most likely outcome determinization
- Extreme 2
  - all outcome determinization
- Middle ground
  - primary outcome (upto l) determinization

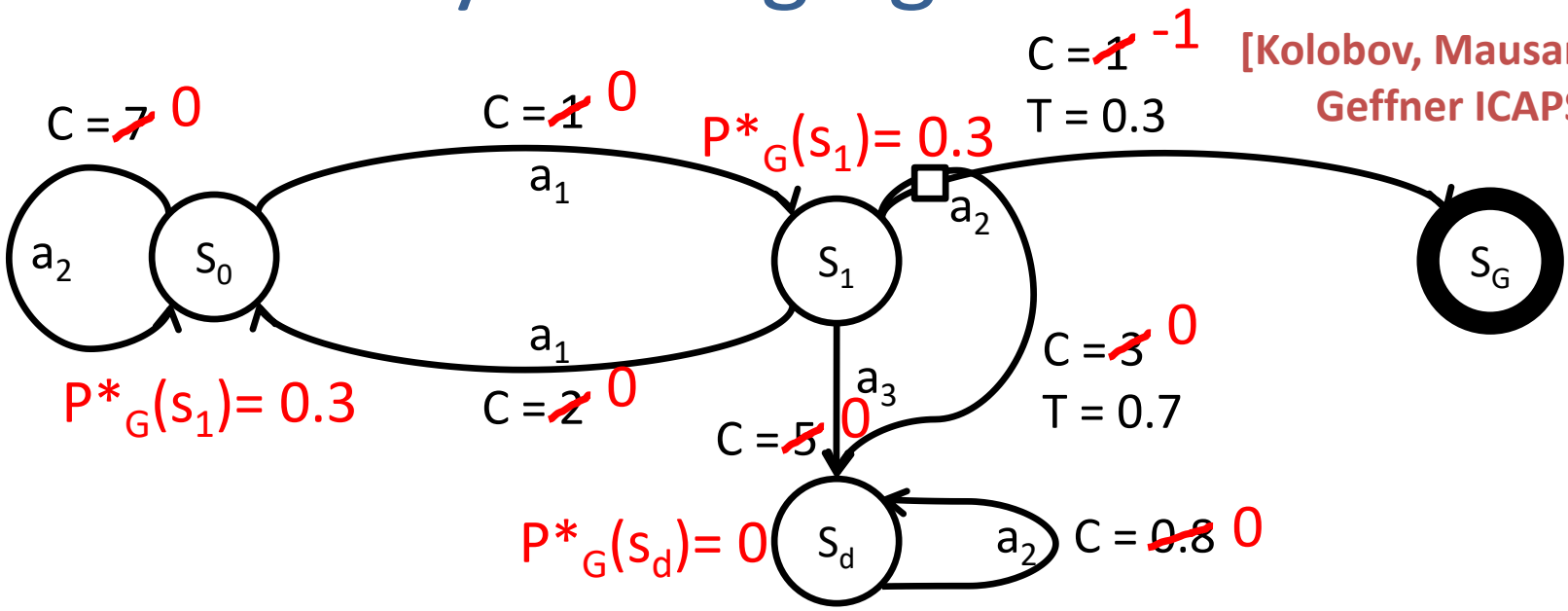
- Extreme 1
  - all actions deterministic
- Extreme 2
  - all actions completely probabilistic
- Middle ground
  - actions have primary outcomes + (upto k) exception outcomes

BEYOND SSPs (contd)

Unavoidable Dead-ends (contd)

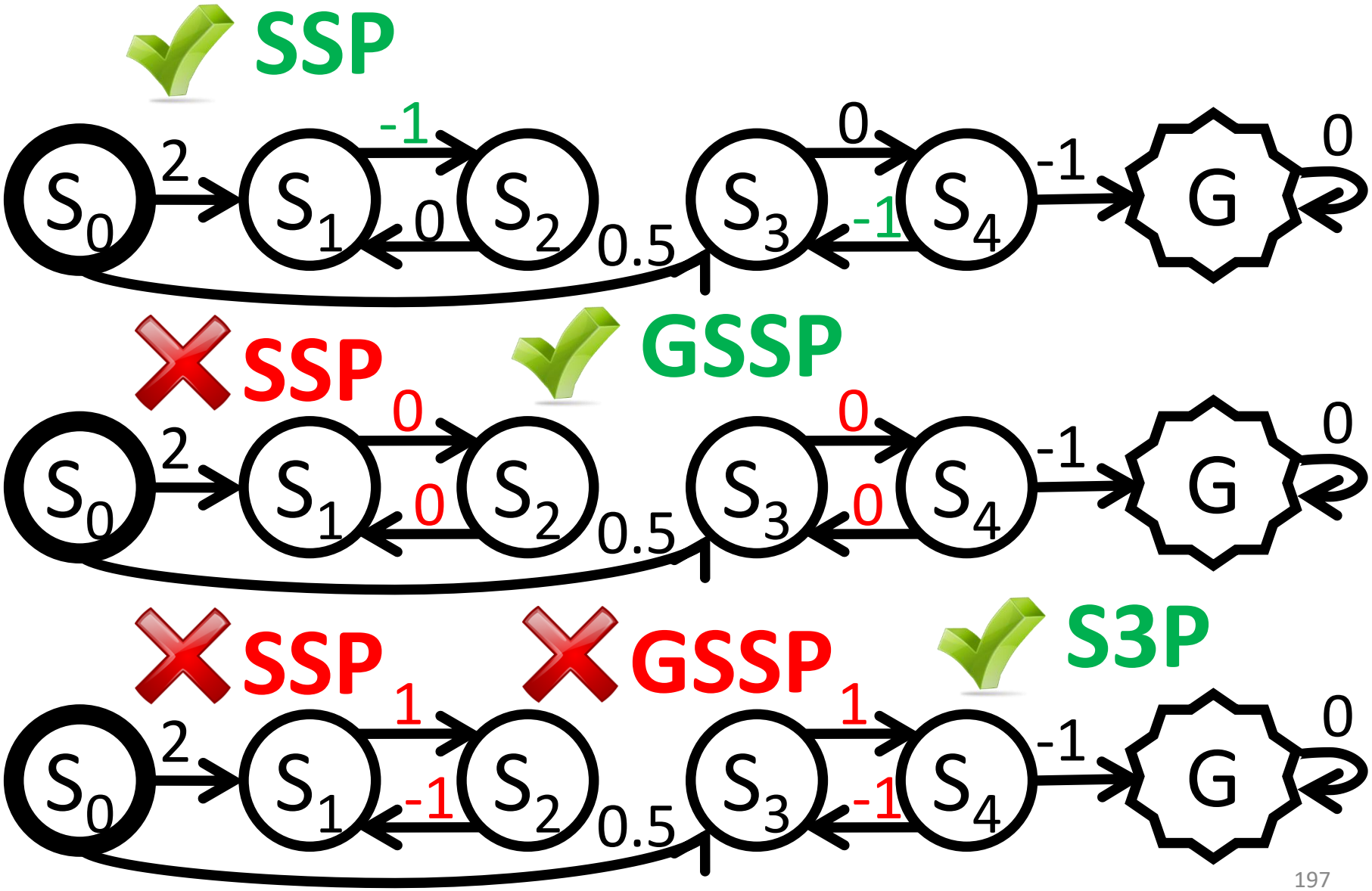
- iSSPUDE: Infinite-Penalty SSP with Unavoidable Dead-Ends [Kolobov et al 12]
  - (MAXPROB) find policy that first maximizes the prob of reaching goal
  - from all such policies find one minimum expected cost
- Dual objective
- iSSPUDE is much harder than SSP_{s0}

MAXPROB: Dealing with Unavoidable Infinitely Damaging Dead Ends-1



- Comparing policies in terms of cost meaningless
- MAXPROB/GSSP MDPs: evaluate policies by probability of reaching goal
  - Set all action costs to 0 (they don't matter), reward 1 for reaching goal
  - Fixed-point methods such as VI or LRTDP don't converge because of traps

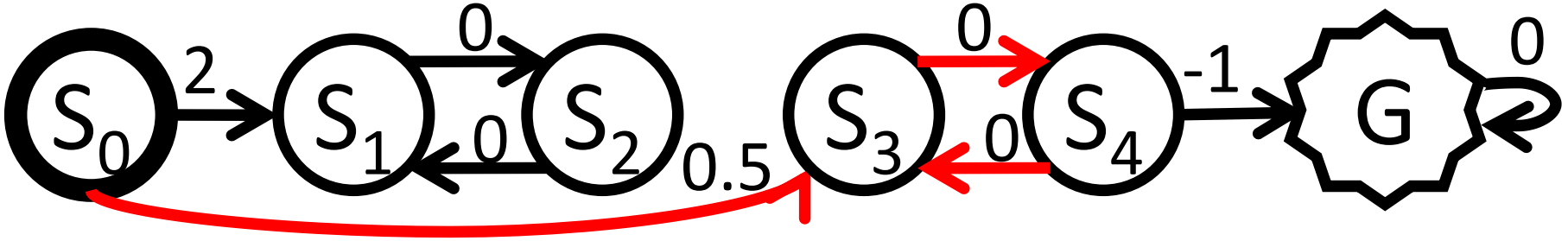
(Maximization) MDP Examples



Generalized SSPs: Example

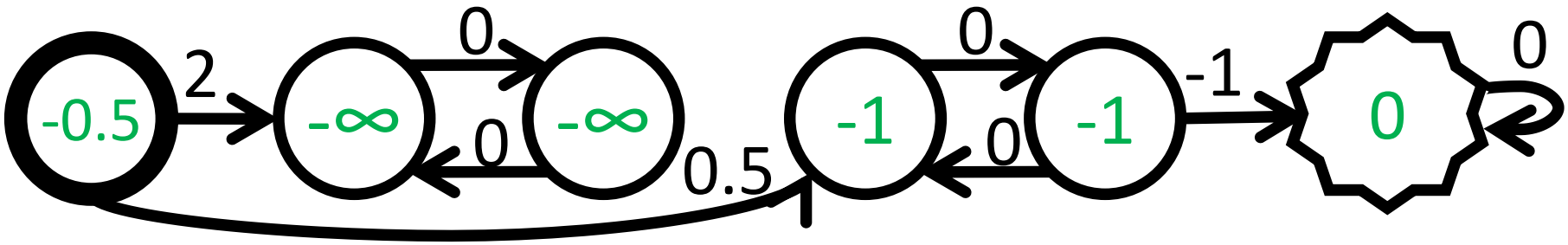
Solution

Not a solution



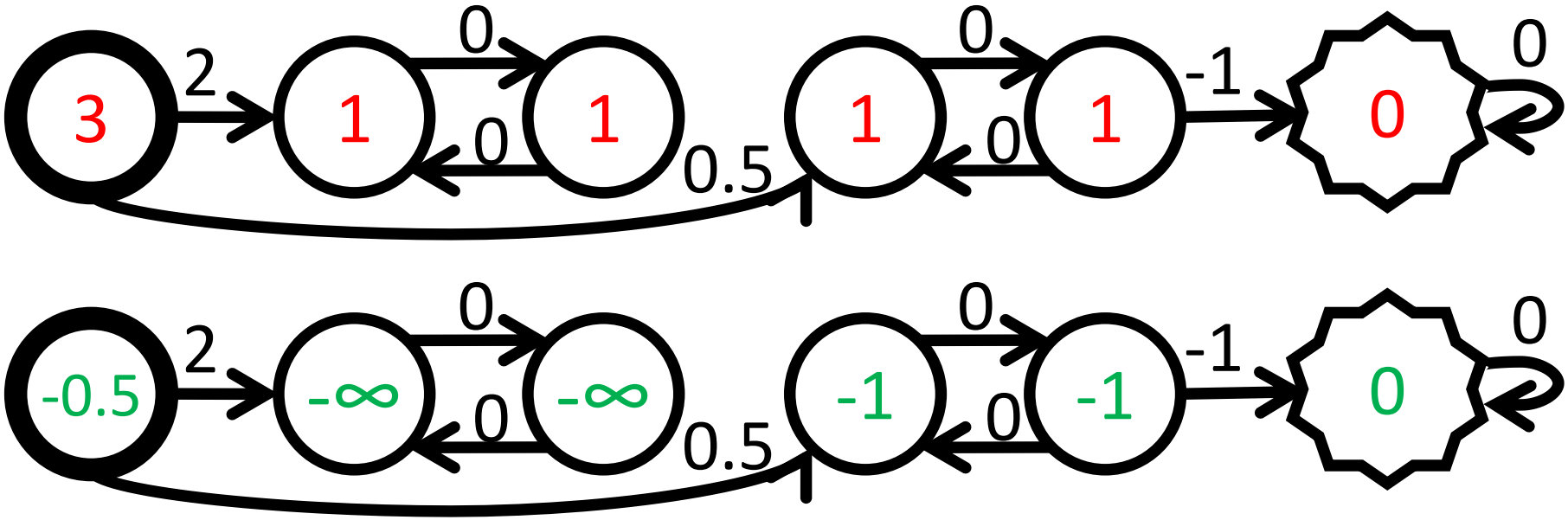
GSSPs: Is V* A Fixed Point of *B*?

- Reminder: in SSPs, V* = *B* V*, where
  - *B* is the *Bellman backup operator*
  - $B\,V(s) = \max_a \{R(s, a) + \sum_{s' \text{ in } succ(s,a)} T(s, a, s')V(s')\}$
- In SSPs, V* is a fixed point of *B*
  - **Still true in GSSPs:**



GSSPs: Is V* *The **Unique*** Fixed Point of *B?*

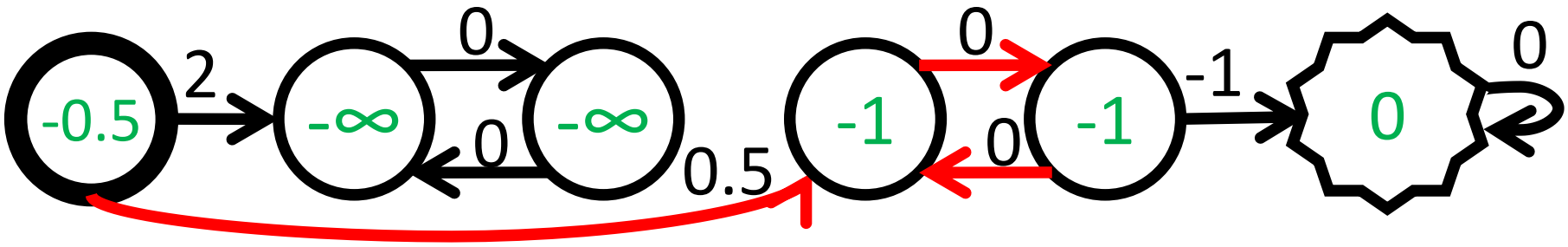
- In SSPs, V* is the unique fixed point of *B*
  - I.e., V* = *B* o *B* o … *B* V_0, V_0 is a *heuristic value function*
  - **Not true in GSSPs:**



  - **Moreover, all suboptimal fixed points are admissible!**

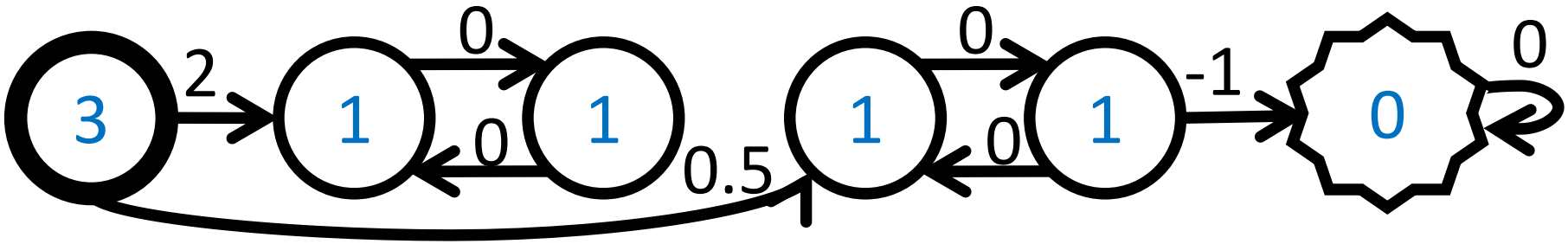
GSSPs: Is Every V*-greedy π A Solution?

- In SSPs, every π greedy w.r.t V* reaches the goal
  - **Not true in GSSPs:**

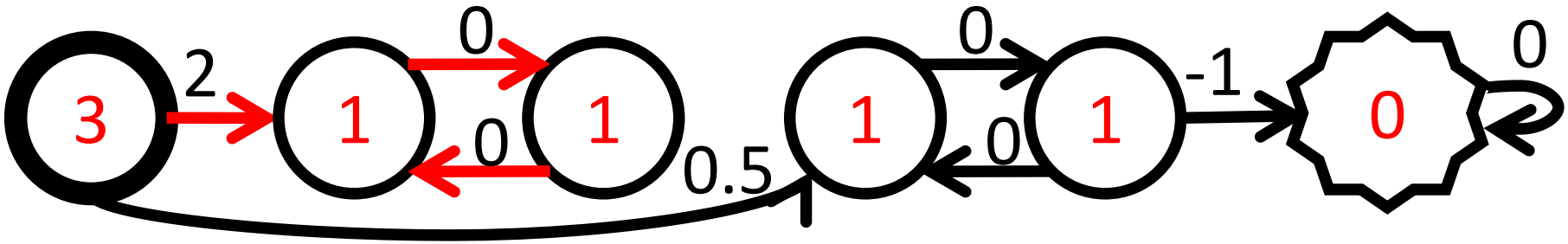


Efficiently Solving GSSPs: Attempt #1

- **Just Run F&R!**
  - Start with an admissible V_0

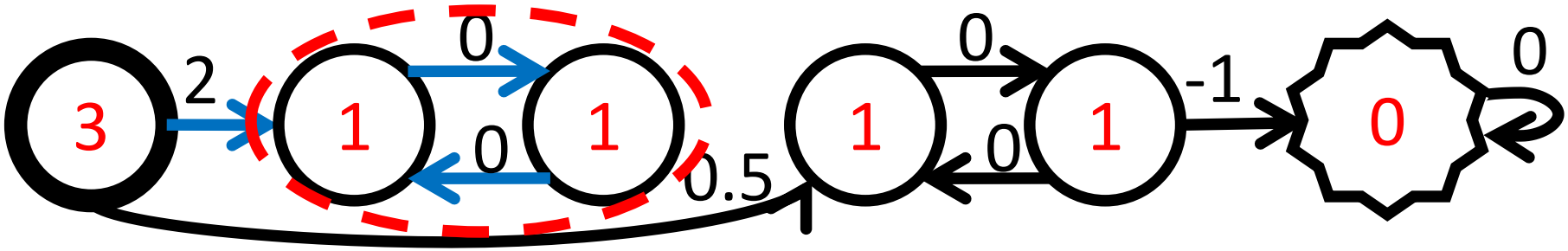


  - Done!



Attempt #1: What Went Wrong?

- In GSSPs, suboptimal fixed points are admissible!
  - When starting with $V_0 \geq V^*$, F&R hit one of them.
  - *B* can't change V over ***traps*** – strongly connected components in V's greedy graph



- Can yield an arbitrarily poor solution

Efficiently Solving GSSPs: **FRET**

- **F**ind, **R**evise, **E**liminate **T**raps
  - First heuristic search algorithm for MDPs beyond SSP
  - Provably optimal if the heuristic is admissible

- Main idea
  - Run F&R until convergence
  - Eliminate traps in the policy envelope
  - Repeat until no more traps

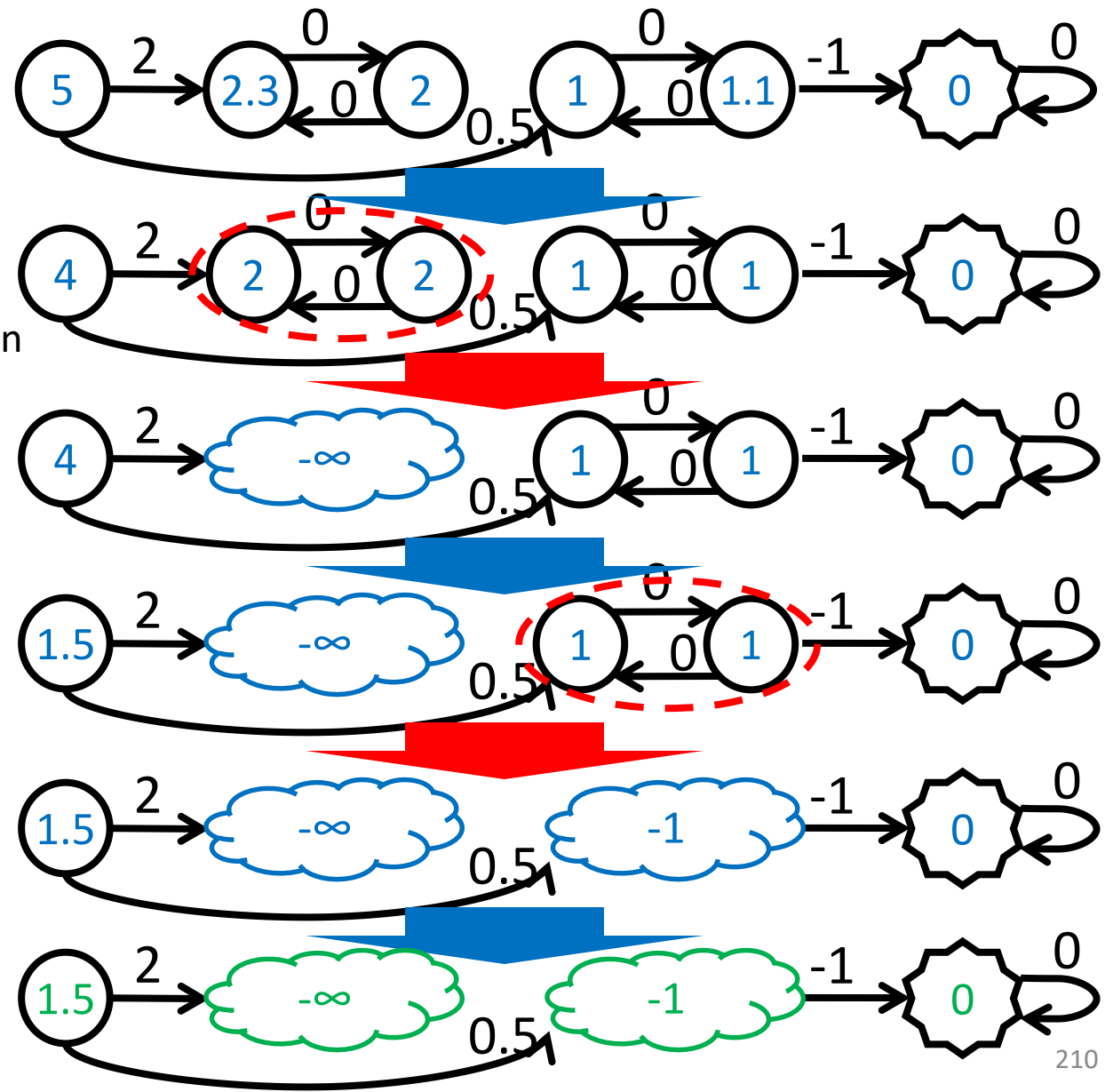
FRET Example: Finding V*

Repeat

Start with an admissible V_0

Run **F&R** until convergence

Eliminate Traps in the resulting V_i



Find-and-Revise

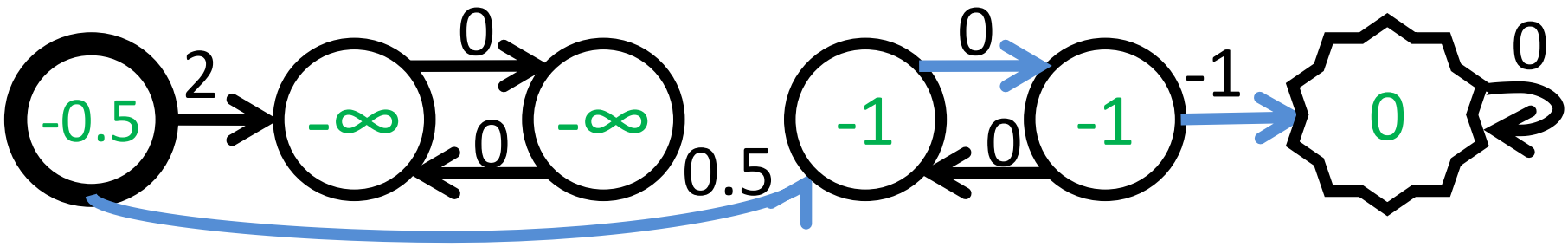
Eliminate Traps

Find-and-Revise

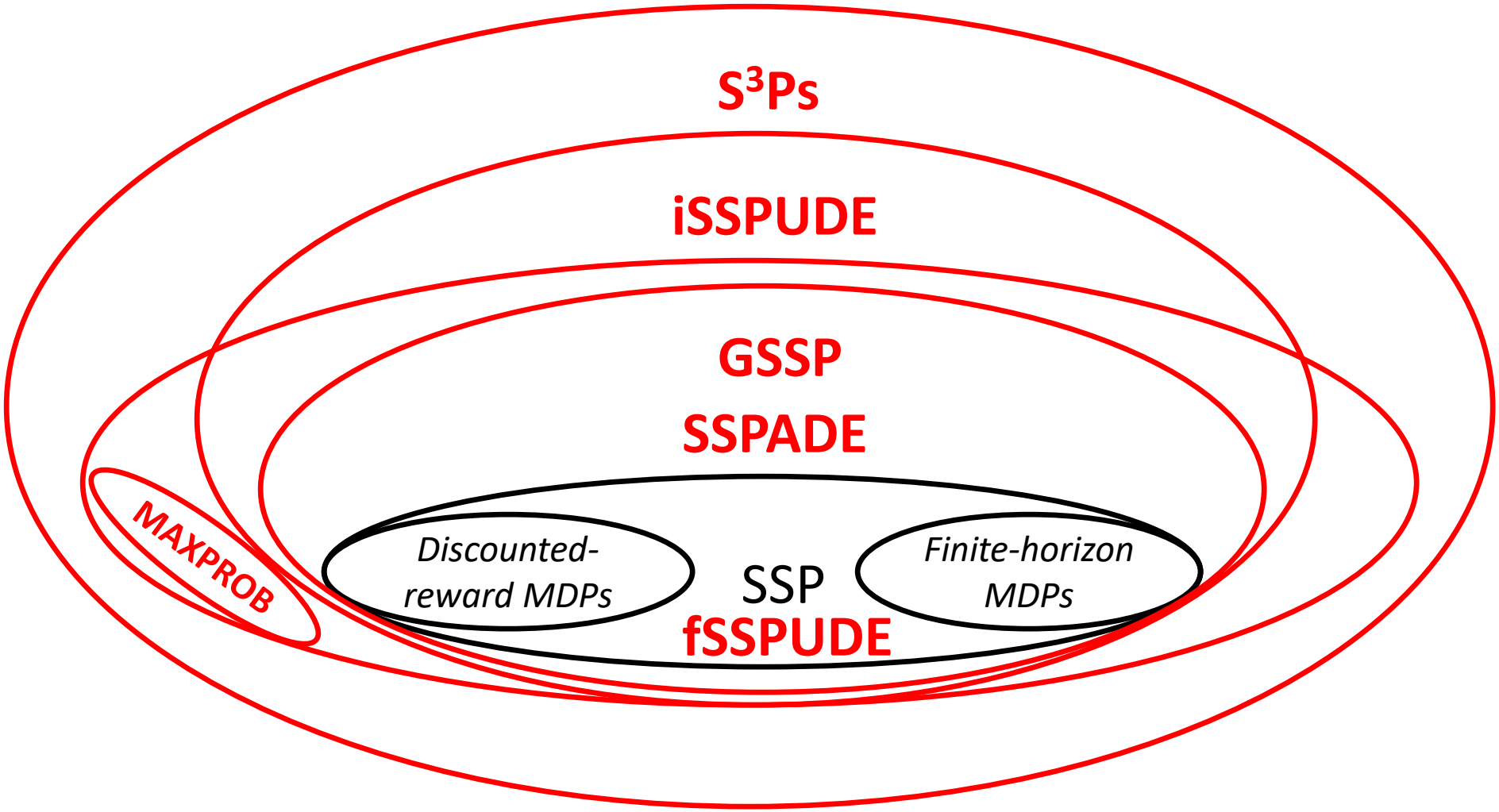
No traps left – done!

FRET Example: Extracting π*

- Iteratively "connect" states to the goals
  - Using optimal actions
  - Until s_0 is connected



Goal-Oriented MDP Hierarchy



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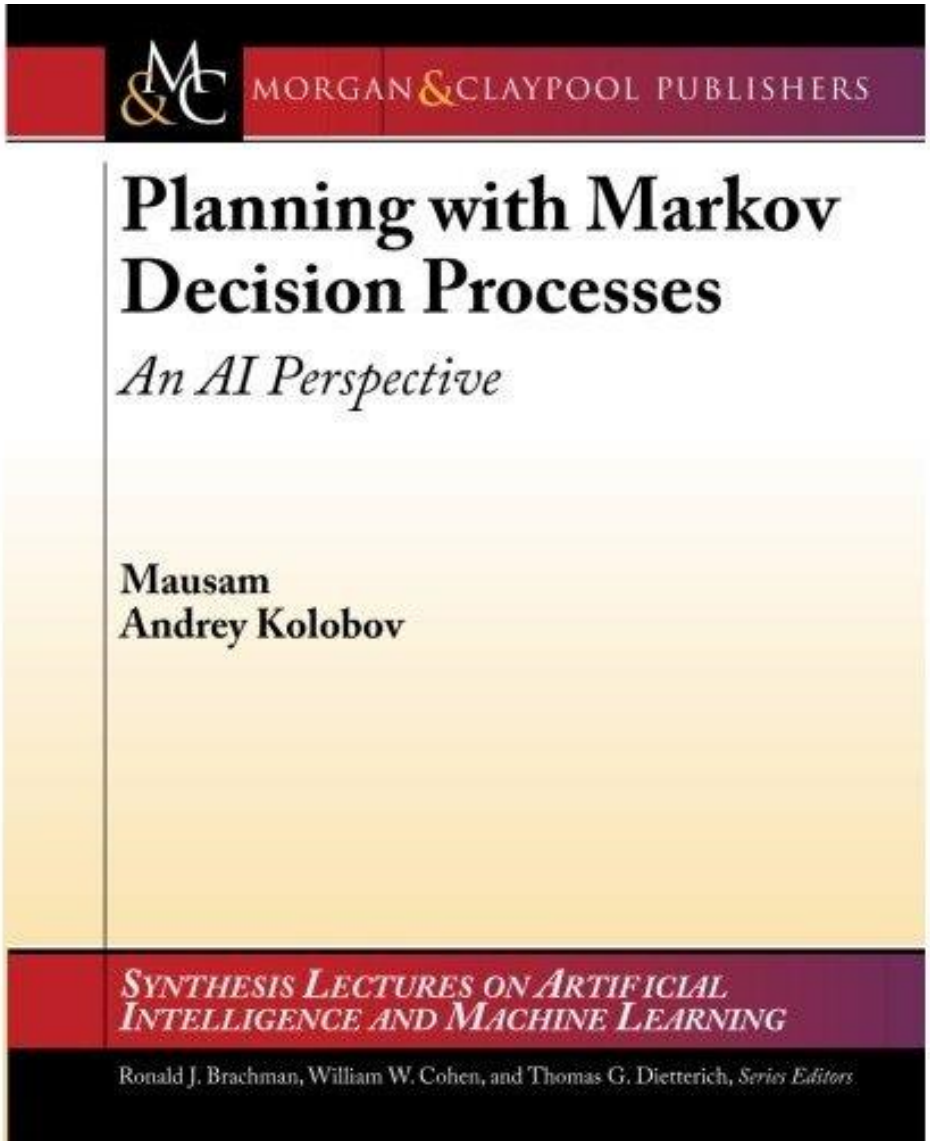
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