MDP Algorithms

Part II

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Outline of this lecture

- Markov decision processes
- Planning via determinization
- Monte-Carlo methods
- Monte-Carlo Tree Search
- Heuristic Search
- Trial-based Heuristic Tree Search

Please ask questions at any time!

Reminder: First Part

- presented several algorithms for probabilistic planning:
  - Determinizations
  - Hindsight Optimization
  - Policy Sampling
  - Sparse Sampling
- there are applications for all where they perform well
- but all are suboptimal even if provided with unlimited resources

Foreword

- there is a lot of research in MCTS $\Rightarrow$ I can only cover the basics
- definition of MCTS controversial in the literature
- I only consider algorithms as MCTS that incorporate concepts of Monte-Carlo and Tree Search

Browne et al.
A Survey of Monte Carlo Tree Search Methods
2012

Idea

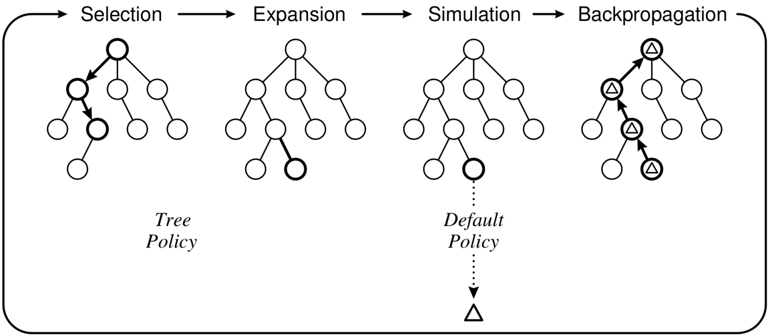
- perform iterations as long as resources (deliberation time, memory) allow:
  - build a partial MDP tree, where nodes n are annotated with
    - average reward $\hat{Q}^{MCTS}k(n.\text{state}, a)$ for action a after k iterations
    - visit counter $N_k(n)$ and $N_k(n, a)$ for action a after k iterations
  - initially, the tree contains only the root node
  - each iteration adds one node to the tree
- execute the action with the highest reward estimate

Iterations

Each iteration consists of (up to) four phases:

- selection: traverse the tree by sampling the execution of the tree policy until
  - a terminal node is reached (skip next two phases and proceed with $r = 0$ in this case), or
  - a state s is reached that is not yet in the tree
- expansion: add a node for s to the tree
- simulation: from s, apply default policy until a terminal state is reached, yielding reward r
- backpropagation: is a has been selected in n, update n:
  - increase visit counter $N_k(n)$ and $N_k(n,a)$
  - set $r \leftarrow r + R(n.\text{state}, a)$
  - update current average reward $\hat{Q}_k^{MCTS}(n,a)$ based on r

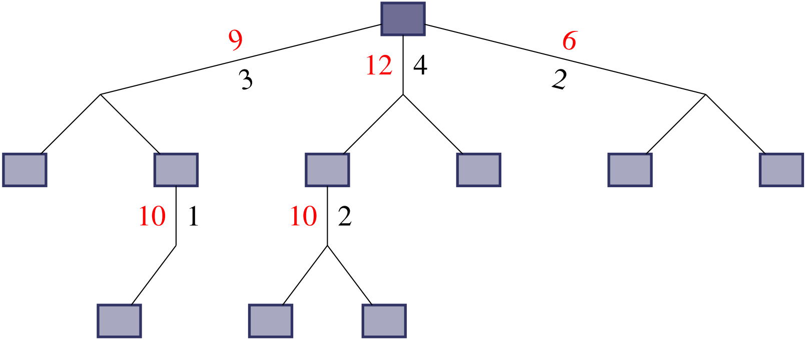
Iterations



Taken from Browne et al., “A Survey of Monte Carlo Tree Search Methods”, 2012

Selection Phase

apply tree policy to traverse tree



Selection Phase

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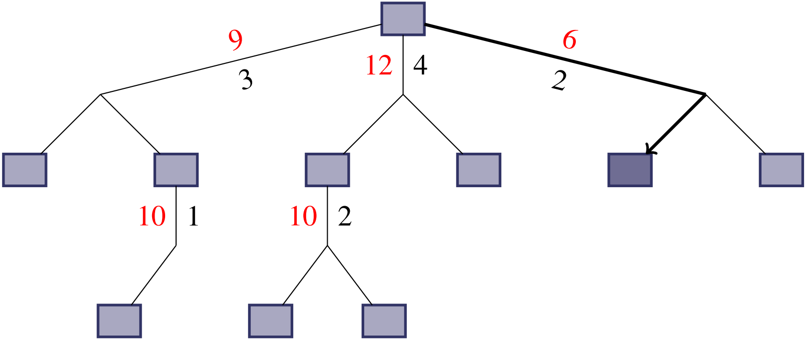

Selection Phase

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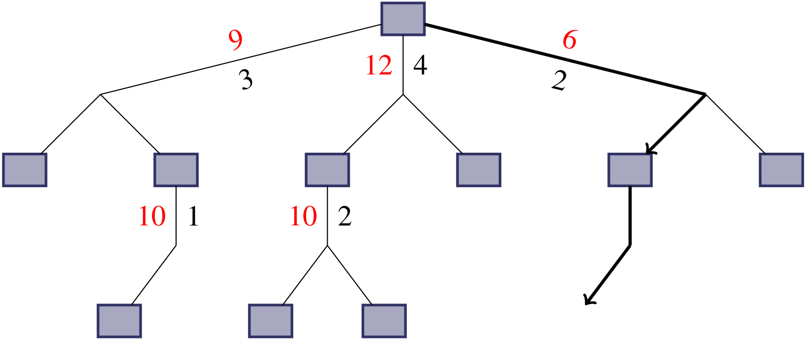
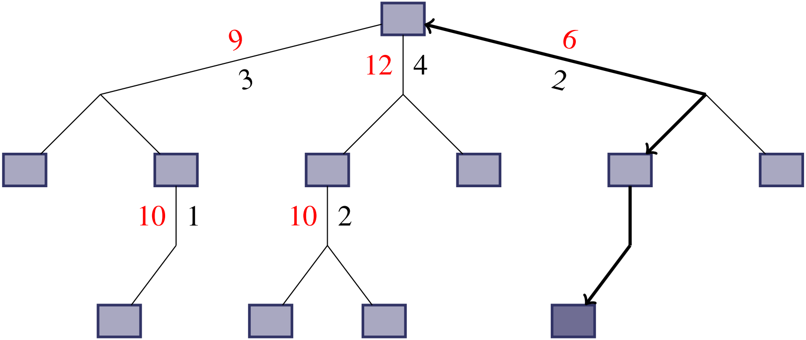


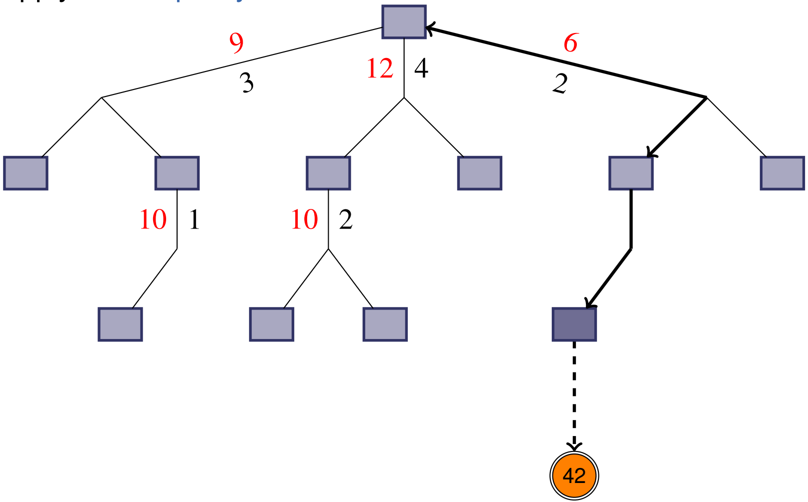
Expansion Phase

create a node for first state not in tree



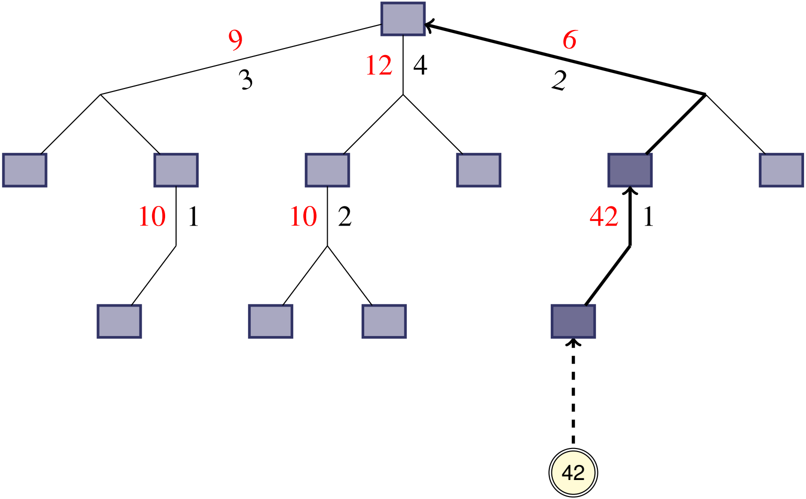
Simulation Phase

apply default policy until terminal state is reached



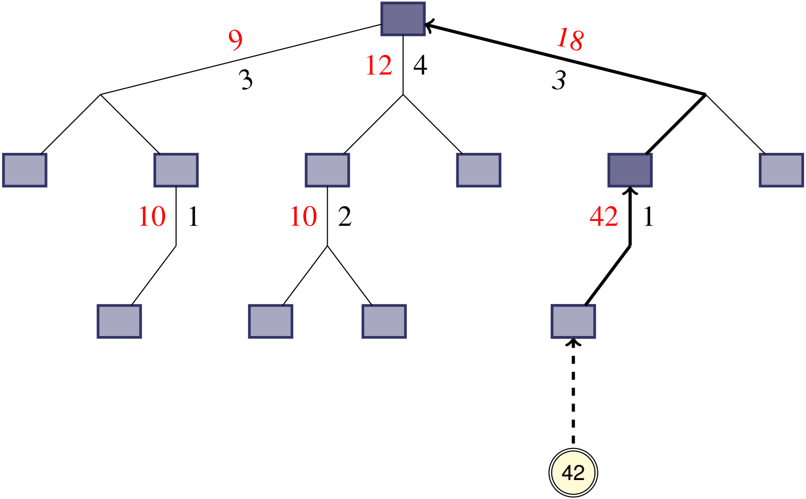
Backpropagation Phase

update visited nodes



Backpropagation Phase

update visited nodes



MCTS Configurations

- MCTS is a framework of algorithms
- concrete algorithms are specified in terms of:
  - tree policy
  - default policy
- for many probabilistic planning problems, there is a well-suited MCTS configuration
- However: there are also many poorly suited MCTS configurations
- and for every MCTS configuration that works well in one problem, there is another problem where it performs poorly

$\Rightarrow$ Are there properties we can analyse theoretically?

Asymptotic Optimality

Asymptotic Optimality

An MCTS algorithm is asymptotically optimal if $Q_k^{MCTS}(n)$ converges to optimal action-value $Q^\star(n.\text{state})$ for all $n \in \text{succ}(n_0)$ with $k \to \infty$.

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An MCTS algorithm is asymptotically optimal if $Q_k^{MCTS}(n)$ converges to optimal action-value $Q^\star(n.\text{state})$ for all $n \in \text{succ}(n_0)$ with $k \to \infty$.

Note: there are MCTS instantiations that act optimally although the values do not converge in this way (e.g., if all $Q_k^{MCTS}(n)$ converge to $\ell \cdot Q^\star(n.\text{state})$ for a constant $\ell > 0$)

Asymptotic Optimality

A tree policy is asymptotically optimal if

- it explores forever:
  - every position is expanded eventually and visited infinitely often
- and it is greedy in the limit:
  - the probability that an optimal move action selected converges to 1

$\rightsquigarrow$ in the limit, backups based on iterations where only an optimal policy is followed dominate suboptimal backups

Tree Policy: Objective

tree policies have two contradictory objectives:

- explore parts of the game tree that have not been investigated thoroughly
- exploit knowledge about good moves to focus search on promising areas

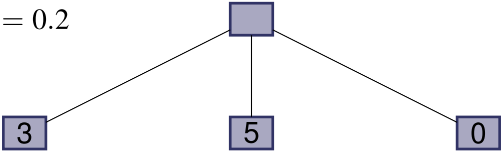
central challenge: balance exploration and exploitation

ε-greedy: Idea

- tree policy with constant parameter ε
- with probability $1 - \varepsilon$, pick a greedy move (i.e., one that leads to a successor node with the best action-value estimate)
- otherwise, pick a non-greedy successor uniformly at random

ε-greedy: Example

$\varepsilon = 0.2$



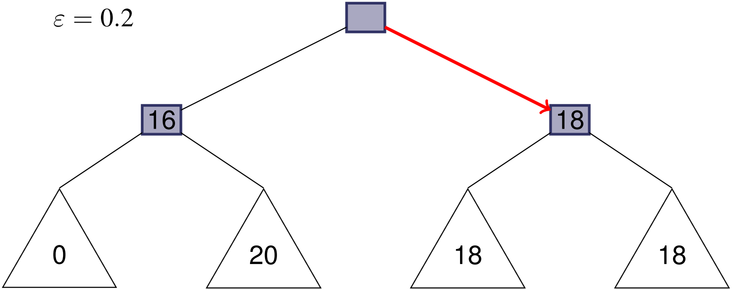
$\mathbb{P}(n_1) = 0.1$ $\mathbb{P}(n_2) = 0.8$ $\mathbb{P}(n_3) = 0.1$

ε-greedy: Asymptotic Optimality

Asymptotic Optimality of ε-greedy

- explores forever
- not greedy in the limit

$\rightsquigarrow$ not asymptotically optimal



ε-greedy: Asymptotic Optimality

Asymptotic Optimality of ε-greedy

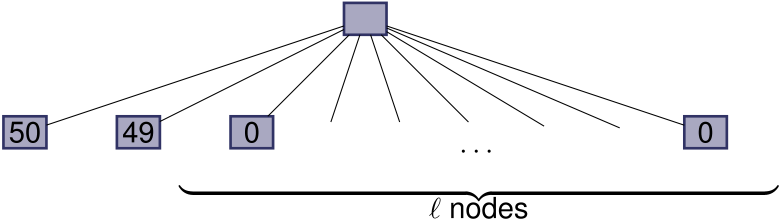
- explores forever
- not greedy in the limit

$\rightsquigarrow$ not asymptotically optimal

asymptotically optimal variant: use decaying ε, e.g. $\varepsilon = \frac{1}{k}$

ε-greedy: Weakness

Problem:
when ε-greedy explores, all non-greedy moves are treated equally



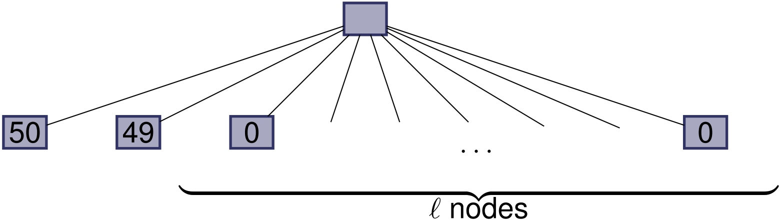
e.g., $\varepsilon = 0.2, \ell = 9$: $\mathbb{P}(n_1) = 0.8$, $\mathbb{P}(n_2) = 0.02$

Softmax: Idea

- tree policy with constant parameter τ
- select moves proportionally to their action-value estimate
- Boltzmann exploration selects moves proportionally to

$$\mathbb{P}(n) \propto e^{\frac{\hat{Q}_k^{\text{MCTS}}(n)}{\tau}}$$

Softmax: Example



e.g., $\tau = 10, \ell = 9$: $\mathbb{P}(n_1) \approx 0.51$, $\mathbb{P}(n_2) \approx 0.46$

Boltzmann Exploration: Asymptotic Optimality

Asymptotic Optimality of Boltzmann Exploration

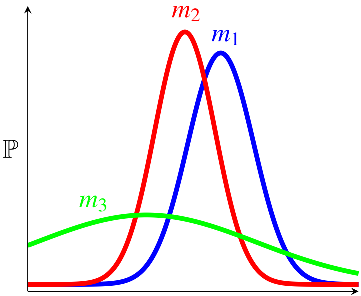
- explores forever
- not greedy in the limit
  (probabilities converge to positive constant)

$\rightsquigarrow$ not asymptotically optimal

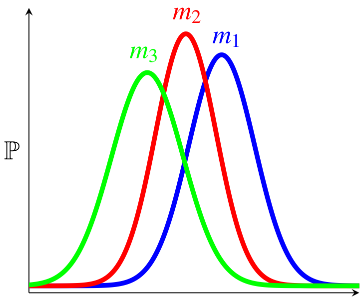
asymptotically optimal variants: use decaying τ

careful: τ must not decay faster than logarithmically
(i.e., must have $\tau \geq \frac{\text{const}}{\log k}$) to explore infinitely

Boltzmann Exploration: Weakness



scenario 1: high variance for m_3



scenario 2: low variance for m_3

- Boltzmann exploration only considers mean of sampled action-values for the given moves
- as we sample the same node many times, we can also gather information about variance (how reliable the information is)
- Boltzmann exploration ignores the variance, treating the two scenarios equally

Upper Confidence Bounds: Idea

balance exploration and exploitation by preferring moves that

- have been successful in earlier iterations (exploit)
- have been selected rarely (explore)

Upper Confidence Bounds: Idea

Upper Confidence Bounds

- select successor n' of n that maximizes $\hat{Q}_k^{MCTS}(n') + B(n')$
- based on action-value estimate $\hat{Q}^{\text{MCTS}}(n')$
- and a bonus term $B(n')$
- select $B(n')$ such that $Q^\star(n'.\text{state}) \leq \hat{Q}_k^{MCTS}(n') + B(n')$ with high probability
- idea: $\hat{Q}_{(}^{\text{MCTS}}n') + B(n')$ is an upper confidence bound on $Q^\star(n')$ under the collected information

Upper Confidence Bounds: UCB1

- use $B(a) = \sqrt{\frac{2 \cdot \ln N_k(n)}{N_k(n',a)}}$ as bonus term
- bonus term is derived from Chernoff-Hoeffding bound:
  - gives the probability that a sampled value (here: $\hat{Q}_k^{\text{MCTS}}(n')$)
  - is far from its true expected value (here: $Q^\star(n'.\text{state})$)
  - in dependence of the number of samples (here: $N_k(n', n'.\text{action})$)
- picks the optimal move exponentially more often

Upper Confidence Bounds: Asymptotic Optimality

Asymptotic Optimality of UCB1

- explores forever
- greedy in the limit

$\rightsquigarrow$ asymptotically optimal

Upper Confidence Bounds: Asymptotic Optimality

Asymptotic Optimality of UCB1

- explores forever
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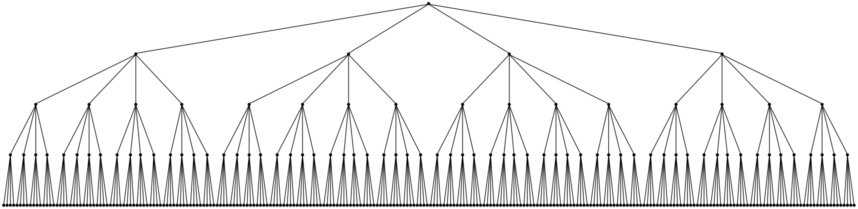
However:

- no theoretical justification to use UCB1
  in trees or planning scenarios
- development of tree policies active research topic

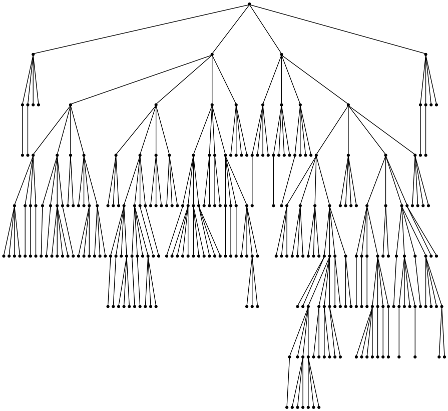
Tree Policy: Asymmetric Game Tree

full tree up to depth 4



Tree Policy: Asymmetric Game Tree

UCT tree (equal number of search nodes)



Default Policy

- Most often used: Monte-Carlo random walk
- Very successful: learned default policies, (e.g., for Go) Requires large amounts of data and/or computational power

AO*: Idea

AO* for acyclic MDPs (Nilsson, 1971)

- add s_0 to open list and to greedy policy graph
- repeat until greedy policy graph has no fringe
  - expand best state s on the fringe of the greedy graph
  - initialize all new states with heuristic
  - backup all states from where s is reachable
  - recompute the greedy graph
- output the greedy graph as the final policy

AO* Tree Search

- perform iterations as long as resources (deliberation time, memory) allow:
- build a partial MDP tree, where nodes n are annotated with
  - average reward
  - a solve label
- initially, the tree contains only the root node
- each iteration expands one node in the tree

AO* Tree Search: Iterations

Iterations proceed as follows:

- starting from the root node, select a successor node with maximum action-value estimate
- until a node is reached without successors
- expand that node and add all children to the tree
- initialize action-value estimate of all children with heuristic
- if a child is a terminal node, set its solve label to true
- if a has been selected in n, update n:
  - derive successor n' with maximum action value
  - set $\hat{Q}_k(n) = \hat{Q}_k(n') + R(n.\text{state}, n'.\text{action})$
  - set the solve label of n to the value of the solve label of n'

Real-Time Dynamic Programming

- AO* Tree Search very similar to (Trial-based) Real-Time Dynamic Programming (Barto, Bradtke, and Singh 1995)
- Difference: RTDP expands states until a terminal state is reached

Trial-based Heuristic Tree Search

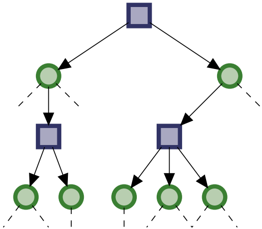
asymptotically optimal MDP algorithms:

- Monte-Carlo Tree Search (e.g., UCT)
- Dynamic Programming (e.g., RTDP)
- Heuristic Search (e.g., AO*)

all have complementary strengths
common framework that allows to describe them all Trial-based Heuristic Tree Search (THTS)

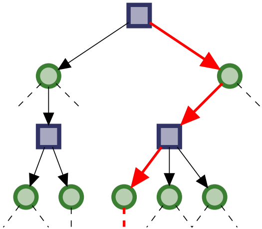
Trial-based Heuristic Tree Search

- initialize search tree with initial state
- extend search tree iteratively in trials



Trial-based Heuristic Tree Search

- initialize search tree with initial state
- extend search tree iteratively in trials
- 6 variable ingredients:
  - action selection
  - outcome selection



Trial-based Heuristic Tree Search

- initialize search tree with initial state
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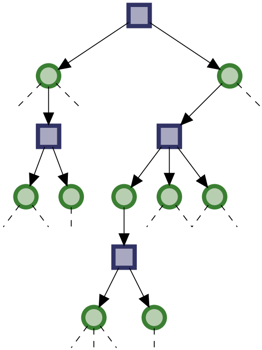
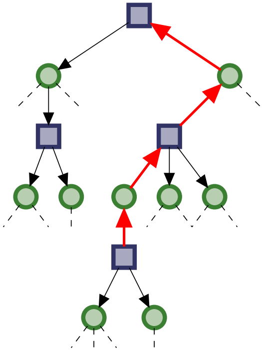
  - initialization
  - trial length



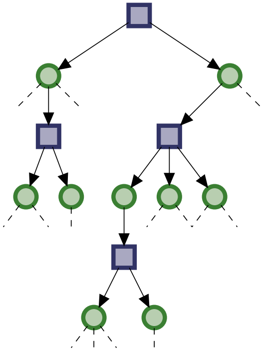
Trial-based Heuristic Tree Search

- initialize search tree with initial state
- extend search tree iteratively in trials
- 6 variable ingredients:
  - action selection
  - outcome selection
  - initialization
  - trial length
  - backup function



Trial-based Heuristic Tree Search

- initialize search tree with initial state
- extend search tree iteratively in trials
- 6 variable ingredients:
  - action selection
  - outcome selection
  - initialization
  - trial length
  - backup function
  - recommendation function



MCTS in the THTS Framework

- trial length: terminate trial once node is added to tree
- action selection: tree policy (e.g., UCB1 for MCTS)
- outome selection: sample
- initialization: add single node to the tree and initialize with heuristic that simulates the default policy
- backup function: Monte-Carlo backups
- recommendation function: expected best arm

AO* in the THTS Framework

- trial length: terminate trial once node is added to tree
- action selection: greedy
- outome selection: sample (or whatever is done in your version of AO*)
- initialization: add all successors, including all outcomes, and initialize with admissible heuristic
- backup function: Bellman backups
- recommendation function: expected best arm

RTDP in the THTS Framework

- trial length: finish trials only in terminal states
- action selection: greedy
- outome selection: sample
- initialization: add all successors, including all outcomes, and initialize with admissible heuristic
- backup function: Bellman backups
- recommendation function: expected best arm

Other Ingredients

- recommendation function:
  - most played arm (MCTS variants)
- initialization:
  - tree expansion: add all (action) successors (UCT*)
  - state-action heuristic, e.g., IDS-based lookahead heuristic of PROST (UCT*)

Other Ingredients: backup functions

- Temporal Differences
- Q-Learning
- MaxMonte-Carlo (DP-UCT)
- Partial Bellman (UCT*)

Considered ingredients in my dissertation

- 1 trial length, 1 outcome selection, 1 initialization
- 2 different recommendation functions
- 9 different backup functions
- 9 different action selections

$\Rightarrow$ 162 different THTS algorithms

115 asymptotically optimal

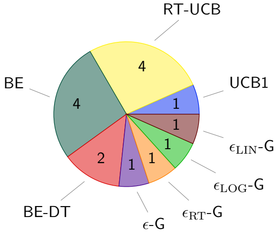
Experimental Evaluation

- most played arm recommendation function often better than same configuration with expected best arm

	Academic	Crossing	Elevators	Game	Navigation	Recon	Skill	Sysadmin	Tamarisk	Traffic	Triangle	Wildfire	**Total**
$\mathrm{MC}^{\mathrm{UCB1}}_{\mathrm{MPA}}$	27	65	78	86	45	92	77	89	86	71	46	84	**70**
Prost 2011	26	62	49	84	42	90	69	88	83	60	49	85	**66**

Experimental Evaluation

- most played arm recommendation function often better than same configuration with expected best arm
- Boltzman exploration and root-valued UCB1 perform best in most domains



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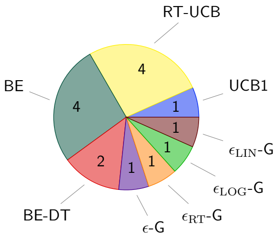
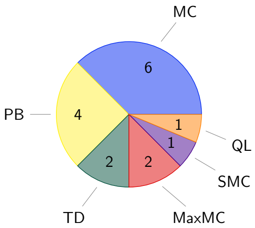
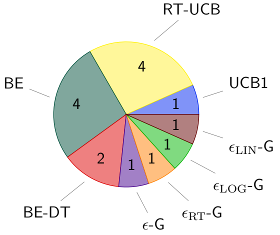
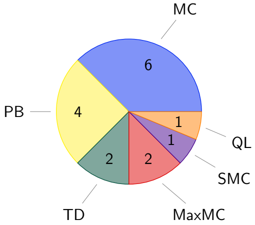




Experimental Evaluation

- most played arm recommendation function often better than same configuration with expected best arm
- Boltzman exploration and root-valued UCB1 perform best in most domains
- Monte-Carlo and Partial Bellman backups perform best in most domains
- almost all action selections and backup functions perform best in at least one domain





Implementation: PROST

- The PROST planner is an implementation of the MCTS framework
- mixing and matching of ingredients very simple
- to add new ingredients, just inherit from the corresonding class

https://bitbucket.org/tkeller/prost/

Summary

- MCTS asymptotically optimal with right components
- AO* and RTDP have complementary strength
- THTS allows to combine ideas from MCTS, DP and Heuristic Search
- almost all ingredients perform well in at least one domain